Séance du 13/10/19.

Fin du 44 du TD sur les complexes.

Déf° d'une suite géométrique
* $V_{n+1} = q \times V_n$.

2) Soit $r_n = |z_n|$. *

$r_{n+1} = |z_{n+1}|$

$r_{n+1} = \left|\frac{1+i}{2} \times z_n\right|$ *

* $|z_1 \times z_2| = |z_1| \times |z_2|$

$r_{n+1} = \left|\frac{1+i}{2}\right| \times |z_n|$.

* $|a+ib| = \sqrt{a^2+b^2}$.

$r_{n+1} = \left|\frac{1}{2} + \frac{1}{2} i\right| \times r_n$. *

$r_{n+1} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \times r_n$.

$r_{n+1} = \sqrt{\frac{1}{4} + \frac{1}{4}} \times r_n$

$r_{n+1} = \sqrt{\frac{1}{2}} \times r_n$ *

$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$r_{n+1} = \frac{1}{\sqrt{2}} \times r_n$

$r_{n+1} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} \times r_n$.

$\boxed{r_{n+1} = \frac{\sqrt{2}}{2} \times r_n.}$ Donc (r_n) est bien géométrique de raison $\frac{\sqrt{2}}{2}$.

Or $-1 < \frac{\sqrt{2}}{2} < 1$ donc $\lim\limits_{n \to +\infty} r_n = 0$. donc (r_n) est bien convergente.

Or on se souvient que $r_n = OA_n$. donc la distance OA_n tend vers 0 lorsque n devient grand.

$$L_n = A_0A_1 + A_1A_2 + \dots + A_{n-1}A_n.$$

3)a) $AB = |z_B - z_A|$. Montrons que $A_nA_{n+1} = r_{n+1}$.

Soit $n \in \mathbb{N}$, $A_nA_{n+1} = |z_{n+1} - z_n|$

$$= \left|\frac{1+i}{2}z_n - z_n\right| = \left|\left(\frac{1+i}{2} - 1\right)z_n\right|$$

$$= \left|\frac{1+i}{2} - 1\right| \times |z_n|.$$

$$= \left|\frac{1}{2} + \frac{1}{2}i - 1\right| \times r_n$$

$$= \left|-\frac{1}{2} + \frac{1}{2}i\right| \times r_n = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \times r_n.$$

$$= \frac{\sqrt{2}}{2} \times r_n$$

$$\boxed{A_nA_{n+1} = r_{n+1}}$$

b) Soit $n \in \mathbb{N}$, $L_n = A_0A_1 + A_1A_2 + \dots + A_{n-1}A_n.$

$$L_n = r_1 + r_2 + \dots + r_n.$$

$$L_n = r_1 \times \frac{1 - \left(\frac{\sqrt{2}}{2}\right)^n}{1 - \frac{\sqrt{2}}{2}} \; *$$

(v_n) est géométrique:
$v_0 + v_1 + \dots + v_n$
$= 1^{er}\text{terme} \times \dfrac{1 - q^{\text{nbre de termes}}}{1-q}.$

$$r_1 = |z_1| = |8+8i| = \sqrt{8^2+8^2} = \sqrt{128}$$

$$r_1 = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}.$$

$$L_n = 8\sqrt{2} \times \frac{1 - \left(\frac{\sqrt{2}}{2}\right)^n}{\frac{2-\sqrt{2}}{2}}$$

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$$L_n = \frac{8\sqrt{2} \times 2}{2-\sqrt{2}} \times \left(1-\left(\frac{\sqrt{2}}{2}\right)^n\right).$$

$$L_n = \frac{16\sqrt{2}}{2-\sqrt{2}} \times \left(1-\left(\frac{\sqrt{2}}{2}\right)^n\right).$$

$$L_n = \frac{16\sqrt{2} \times (2+\sqrt{2})}{(2-\sqrt{2}) \times (2+\sqrt{2})} \times \left(1-\left(\frac{\sqrt{2}}{2}\right)^n\right)$$

$$L_n = \frac{32\sqrt{2} + 16 \times 2}{2^2 - \sqrt{2}^2} \times \left(1-\left(\frac{\sqrt{2}}{2}\right)^n\right).$$

$$L_n = \frac{32\sqrt{2} + 32}{2} \times \left(1-\left(\frac{\sqrt{2}}{2}\right)^n\right).$$

$$L_n = (16\sqrt{2} + 16) \times \left(1-\left(\frac{\sqrt{2}}{2}\right)^n\right).$$

c) $\lim\limits_{n \to +\infty} \left(\frac{\sqrt{2}}{2}\right)^n = 0$ donc par somme et produit, on a :

$$\lim_{n \to +\infty} L_n = 16\sqrt{2} + 16.$$

Cours sur les limites:

$\lim f(x)$

$x \to +\infty$

$x \to 3$

$x \to -\infty$

$$\lim_{\substack{x \to 2^+ \\ x > 2}} \mathrm{Ent}(x) = 2$$

$$\lim_{\substack{x \to 2^- \\ x < 2}} \mathrm{Ent}(x) = 1.$$

$$\lim_{x \to 3} 2x+3 = 9.$$

Preuve : $3x^4 - 2x^3 - x^2 + 5x - 1$

$$= x^4\left(3 - \frac{2x^3}{x^4} - \frac{x^2}{x^4} + \frac{5x}{x^4} - \frac{1}{x^4}\right)$$

$$= x^4\left(3 - \frac{2}{x} - \frac{1}{x^2} + \frac{5}{x^3} - \frac{1}{x^4}\right).$$

Or $\lim_{x \to +\infty} \frac{2}{x} = \lim_{x \to +\infty} \frac{1}{x^2} = \lim_{x \to +\infty} \frac{5}{x^3} = \lim_{x \to +\infty} \frac{1}{x^4} = 0.$

donc $\lim_{x \to +\infty} 3x^4 - 2x^3 - x^2 + 5x - 1 = \lim_{x \to +\infty} 3x^4$

Dém : $\lim_{x \to 0} \frac{\sin(x)}{x}$

$$f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}.$$

$$= \lim_{x \to 0} \frac{\sin(x) - \sin(0)}{x - 0}$$

$$= \sin'(0) = \cos(0) = 1.$$

$\to 1.$

$$\lim_{x \to +\infty} \frac{1}{x} \times x = 0 \ ✗$$

Exercice 1

1) $\lim_{x \to +\infty} P(x) = \lim_{x \to +\infty} 5x^3 = +\infty$.

$\lim_{x \to -\infty} P(x) = \lim_{x \to -\infty} 5x^3 = -\infty$

2) $\lim_{x \to +\infty} Q(x) = \lim_{x \to +\infty} -2x^4$

$= -\infty$.

$\lim_{x \to -\infty} Q(x) = \lim_{x \to -\infty} -2x^4 = -\infty$

Exercice 2 :

1) $f(x) = \frac{x^2+3}{1-x}$.

$D_f =]-\infty ; 1[\cup]1 ; +\infty[= \mathbb{R} \setminus \{1\}$.

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