Séance 15/10/19. Suites numériques.

$$\begin{array}{ccccc} & & \mathbb{N} & & \mathbb{R} \\ \text{indice} & \longrightarrow & n & \longmapsto & u_n \quad (u(n)). \end{array}$$

terme.

$$\begin{array}{ccccc} & \mathbb{R} & & \\ \text{Fonct°} : & x & \longmapsto & f(x) \end{array}$$

antécédent

image.

Formule explicite : Ex: $u_n = 2n^2 + 3$.

$u_{10} = 2 \times 10^2 + 3 = 203$.

Formule par récurrence : Ex : $\begin{cases} u_0 = 1 \\ u_{n+1} = 2u_n. \end{cases}$

$u_1 = 2 \times u_0 = 2 \times 1 = 2$.

$u_2 = 2 \times u_1 = 2 \times 2 = 4$.

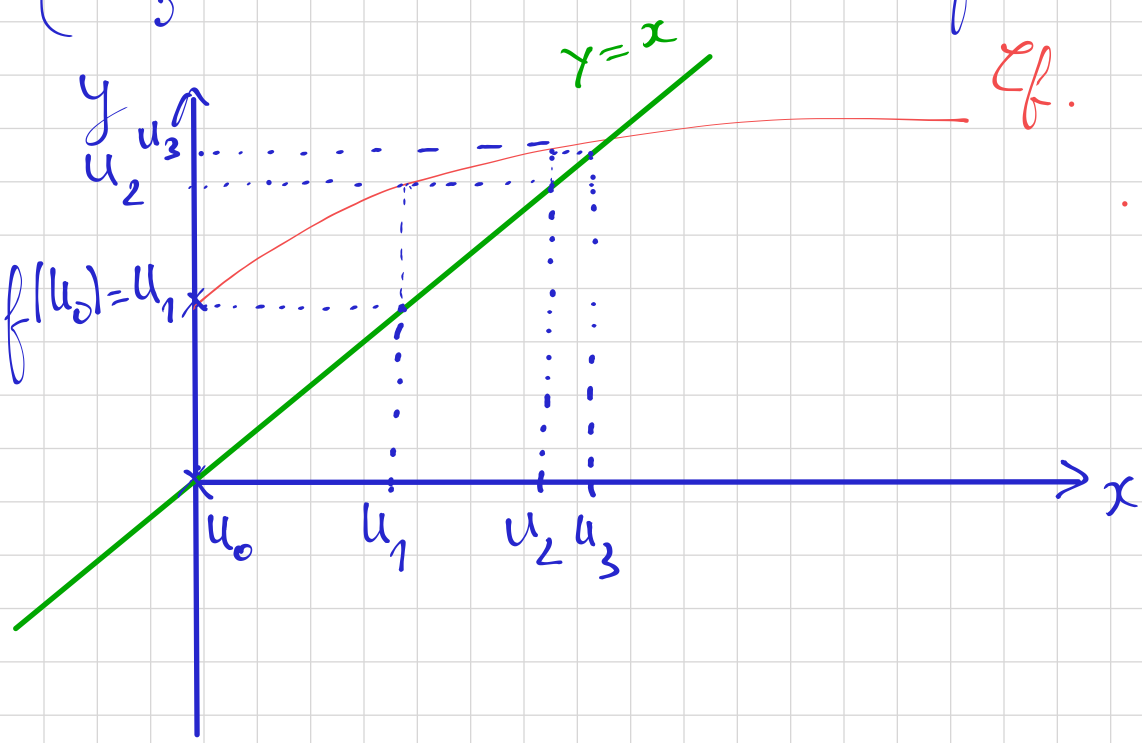
Pour représenter une suite définie explicitement, on fait les étapes suivantes: $u_n = 2n + 3$

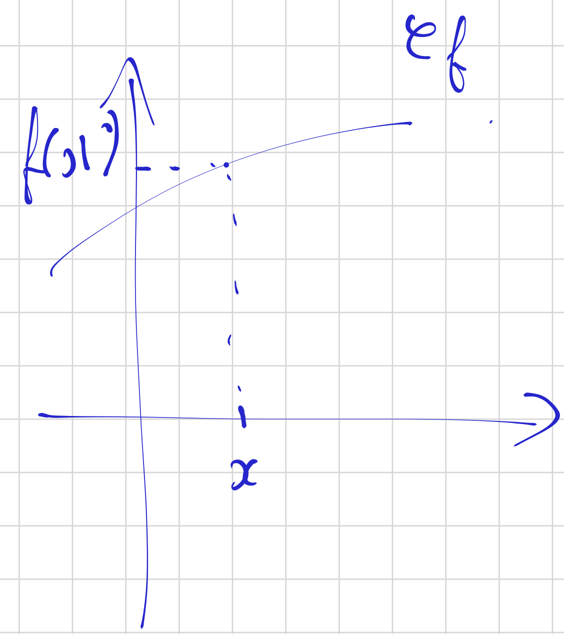
| | | | | | | |
|---|---|---|---|---|---|---|
| $\mathbb{N} \ni n$ | 0 | 1 | ~~1,8~~ | 2 | 3 | 4 |
| $\mathbb{R} \ni u_n$ | 3 | 5 | ~~6~~ | 7 | 9 | 11 |


$$\begin{cases} u_{n+1} = f(u_n) \\ u_0 \text{ est connu.} \end{cases}$$

$u_1 = f(u_0)$.

$u_2 = f(u_1)$.



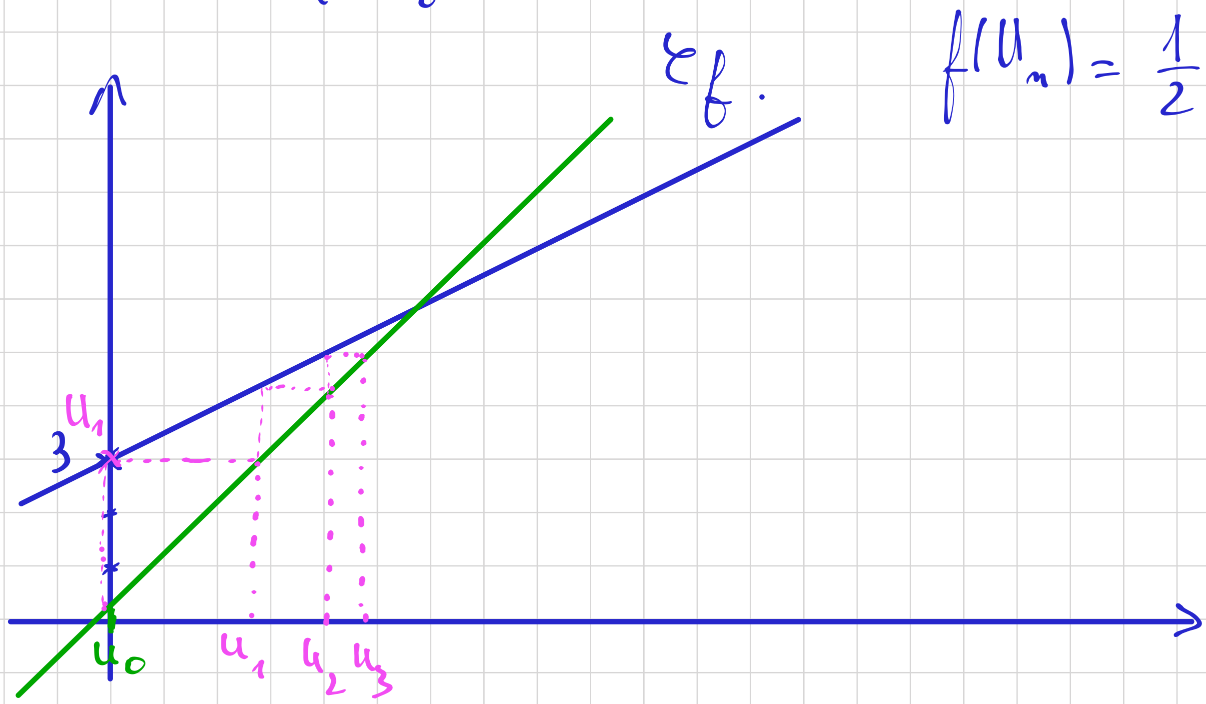




<u>Exemple :</u>

$$\begin{cases} u_{n+1} = \frac{1}{2} u_n + 3. \\ u_0 = 0. \end{cases}$$

$f(x) = \frac{1}{2} x + 3$.

$f(u_n) = \frac{1}{2} u_n + 3 = u_{n+1}$.



$u_{1000} =$

$$u_n = \frac{2n+3}{n+4} = \frac{2\times 1000+3}{1000+4}$$

$$= \frac{2003}{1004}$$

$$= 1{,}99501992.$$

$(u_n)$ :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $u_n$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14. |

+2 +2 +2 +2 +2 +2 +2

$r$ : raison.

$u_n = u_0 + nr.$

$$\boxed{u_n = 5 + 2n.}$$

$u_0 = 5$

$r = 2.$

| | | | | | |
|---|---|---|---|---|---|
| $n$ | 0 | 1 | 2 | 3 | 4 |
| $u_n$ | 5 | 7 | 9 | 11 | 13 |

$$\begin{cases} u_{n+1} = u_n + 3 \\ u_0 = 4 \end{cases}$$

$u_1 = u_0 + 3 = 4+3 = 7.$

$u_2 = u_1 + 3 = 7+3 = 10$

Ex: $u_n = 3n+2.$

$u_{n+1} - u_n = 3(n+1)+2 \quad -(3n+2)$

$= \cancel{3n}+3\cancel{+2} \cancel{-3n}\cancel{-2}$

$= 3.$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $u_n$ | 2 | 5 | 8 | 11 | 14 | 17 | 20 |

$\cancel{2}+5+\cancel{8}+11+\cancel{14}+17+20 =$

$= 16 + 24 + 37$

$= 40+37 = 77.$

$$u_0+u_1+u_2+u_3+u_4+u_5+u_6 = \frac{7\times(2+20)}{2}$$

$$= \frac{14+140}{2} = \frac{154}{2}$$

$$= 77.$$

## Suites géométriques :

$v_0 = 1 \xrightarrow{\times 2}$

$v_1 = 2 \xrightarrow{\times 2}$

$v_2 = 4 \xrightarrow{\times 2}$

$v_3 = 8 \xrightarrow{\times 2}$

$v_4 = 16$

$v_5 = 32 \xrightarrow{\times 2}$

$v_6 = 64 \xrightarrow{\times 2}$

$v_7 = 128 \xrightarrow{\times 2}$

$v_8 = 256 \xrightarrow{\times 2}$

$v_9 = 512$

$v_{10} = 1024.$

$V_n = V_0 \times q^n.$

$V_n = 2 \times 3^n.$

$V_0 = 2 \quad q = 3.$

$V_0 = 2 \times 3^0 = 2 \times 1 = 2$ $\times 3$

$V_1 = 2 \times 3^1 = 2 \times 3 = 6.$ $\times 3.$

$V_2 = 2 \times 3^2 = 2 \times 9 = 18$

$$V_n = 3 \times \left(\frac{1}{2}\right)^n.$$

$ax^2 + bx + c.$

$\underbrace{3}_{V_0} \quad \underbrace{\tfrac{1}{2}}_{q}$

$V_0 = 3 \times \left(\frac{1}{2}\right)^0 = 3.$

$V_1 = 3 \times \left(\frac{1}{2}\right)^1 = \frac{3}{2} = 1{,}5.$

$V_2 = 3 \times \left(\frac{1}{2}\right)^2 = 3 \times \frac{1}{4} = \frac{3}{4}$

$= 0{,}75.$

$U_{n+1} \geq U_n$ suite croissante.

$U_{n+1} \leq U_n$ suite décroissante.

TD sur les suites.

<u>n°3.</u>

1) $U_n = \dfrac{3n-2}{n+1}$ $(n+1)$

$\frac{3 \times 5}{4 \times 5} - \frac{3 \times 4}{5 \times 4}$

$$U_{n+1} - U_n = \frac{3(n+1)-2}{n+1+1} - \frac{3n-2}{n+1}.$$

$$= \frac{3n+3-2}{n+2} - \frac{3n-2}{n+1} = \frac{3n+1}{n+2} - \frac{3n-2}{n+1}$$

$$= \frac{(3n+1)(n+1)}{(n+2)(n+1)} - \frac{(3n-2)(n+2)}{(n+1)(n+2)} = \frac{3n^2+3n+n+1-(3n^2+6n-2n-4)}{(n+1)(n+2)}$$

$$= \frac{\cancel{3n^2}+\cancel{4n}+1-\cancel{3n^2}-\cancel{4n}+4}{(n+1)(n+2)} = \frac{5}{(n+1)(n+2)}.$$

Or $n \in \mathbb{N}$ donc $n+1>0$ et $n+2>0$

donc $(n+1)(n+2)>0$.

Or $5>0$

Par quotient, $\dfrac{5}{(n+1)(n+2)}>0$

Donc $U_{n+1} - U_n > 0$ alors $(U_n)$ est croissante.