Séance du 22/10/19.

TD Sur les suites numériques.

$u_n = u_0 + n r$. explicite.

$\left\{ \begin{array}{l} u_{n+1} = u_n + r \\ u_0 \text{ précisé.} \end{array} \right\}$ formule récursive.

$u_0 + u_1 + u_2 + \ldots + u_n = \text{nombre de termes} \times \dfrac{(1^{er} + d^{er})}{2}$.

n° 6 :

a) $u_0 = 1$; $u_{10} = 31$ Comme (u_n) est arithmétique :

$u_n = u_0 + n \times r$.

$u_n = 1 + n r$.

On $u_{10} = 31$

$1 + 10 \times r = 31$

$10 r = 31 - 1$

$10 r = 30$

$\dfrac{\cancel{10} r}{\cancel{10}} = \dfrac{30}{10}$

$\boxed{r = 3.}$

d'où $u_n = 1 + 3n$.

$u_{2015} = 1 + 3 \times 2015$

$= 1 + 6045$

$= 6046$.

b) $u_0 = 5$ et $u_{100} = -45$. On sait que $u_n = u_0 + n \times r$.

$$u_n = 5 + n \times r.$$

Or on sait que $u_{100} = -45$.

$$5 + 100 \times r = -45.$$

$$100r = -45 - 5.$$

$$100r = -50.$$

$$r = \frac{-50}{100}$$

$$\boxed{r = -\frac{1}{2}}.$$

$$u_n = 5 - \frac{1}{2} \times n.$$

donc $u_{20} = 5 - \frac{1}{2} \times 20 = 5 - 10 = -5$.

c) $u_n = u_0 + n \times r.$

$u_n = u_p + (n-p) \times r.$

$u_n = u_{17} + (n-17) \times r.$

$u_n = 24 + (n-17) \times r.$

Or $u_{40} = 70$

$$24 + (40-17) \times r = 70.$$

$$24 + 23 \times r = 70.$$

$$23r = 70 - 24.$$

$$23r = 46$$

$$r = \frac{46}{23} = \boxed{2.}$$

D'où $u_n = 24 + (n-17) \times 2.$

$u_0 = 24 + (0-17) \times 2.$

$u_0 = 24 - 17 \times 2$

$u_n = u_0 + n r.$

$u_0 = 24 - 34$

$u_0 = -10.$

$u_n = -10 + 2 \times n.$

$u_{40} = -10 + 2 \times 40.$

$u_{17} = -10 + 2 \times 17.$

$u_{40} = -10 + 80 = 70.$

$u_{17} = -10 + 34 = 24.$

n°7.

$$u_{n+1} = \frac{u_n}{1 + u_n}.$$

a) $u_0 = 1.$

$$u_1 = \frac{u_0}{1+u_0} = \frac{1}{1+1} = \frac{1}{2}.$$

$$u_2 = \frac{u_1}{1+u_1} = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} = \frac{1}{3}.$$

$$u_3 = \frac{u_2}{1+u_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{4} = \frac{1}{4}.$$

$$u_4 = (\dots) = \frac{1}{5}.$$

$$u_5 = (\dots) = \frac{1}{6}.$$

b) Soit $n \in \mathbb{N}$, on pose $\boxed{v_n = \frac{1}{u_n}}$

$$v_0 = \frac{1}{u_0} = \frac{1}{1} = 1.$$

$$v_1 = \frac{1}{u_1} = \frac{1}{\frac{1}{2}} = 2.$$

$$v_2 = \frac{1}{u_2} = \frac{1}{\frac{1}{3}} = 3.$$

$$v_3 = \frac{1}{u_3} = \frac{1}{\frac{1}{4}} = 4.$$

$$v_4 = \frac{1}{u_4} = \frac{1}{\frac{1}{5}} = 5.$$

$$v_5 = \frac{1}{u_5} = \frac{1}{\frac{1}{6}} = 6.$$

Pour prouver que (V_n) est arithmétique,

$$\boxed{V_{n+1} = V_n + r.}$$

Or on sait que $V_n = \dfrac{1}{u_n}$.

d'où : $V_{n+1} = \dfrac{1}{u_{n+1}} = \dfrac{\frac{1}{u_n}}{\frac{u_n}{1+u_n}}$] fraction.] fraction.

$\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$.

$$V_{n+1} = 1 \times \frac{1+u_n}{u_n} = \frac{1+u_n}{u_n}.$$

$$V_{n+1} = \frac{1}{u_n} + \frac{u_n}{u_n}.$$

$$\boxed{V_{n+1} = V_n + 1.}$$

Donc (V_n) est arithmétique de raison 1 et de premier terme $V_0 = 1$.

$$V_n = V_0 + n \times r.$$

$$V_n = 1 + n.$$

Or on sait que $V_n = \dfrac{1}{u_n}$.

$$u_n = \frac{1}{V_n}$$

$$\boxed{u_n = \frac{1}{n+1}.}$$

n° 8 :

a) $1 \overset{+2}{+} 3 \overset{+2}{+} 5 + \dots \overset{+2}{+} 99$

$u_n = u_0 + nr.$

$u_n = 1 + n \times 2.$

n	0	1	2	...	49
u_n	1	3	5		99

$u_n = 99.$

$1 + n \times 2 = 99.$

$n \times 2 = 99 - 1.$

$n \times 2 = 98.$

$n = \frac{98}{2} = 49.$

$1 + 3 + 5 + \dots + 99.$

$= u_0 + u_1 + u_2 + \dots + u_{49}$

$= \frac{\text{nbre de termes} \times (1^{\text{er}} \text{ terme} + \text{dernier terme})}{2}$

$= \frac{50 \times (1 + 99)}{2} = \frac{50 \times 100}{2} = 25 \times 100 = 2500 = 50^2.$

b) $S = 1 + 3 + 5 + \dots + (2n - 1).$

n	0	1	2	...	$n-1$
u_n	1	3	5		$2n-1$

$u_n = 1 + 2n.$

$u_{n-1} = 1 + 2 \times (n - 1)$

$= 1 + 2n - 2$

$= 2n - 1.$

$S = \frac{\text{nbre de termes} \times (1^{\text{er}} \text{ terme} + \text{dernier terme})}{2}$

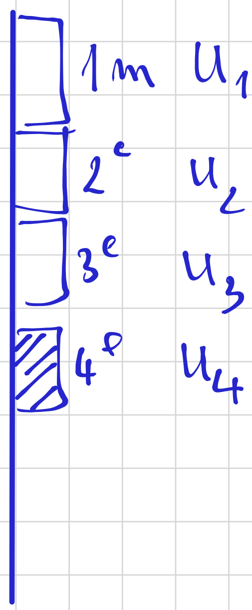
$S = \frac{n \times (1 + 2n - 1)}{2}$

$S = \frac{\cancel{2} n^2}{\cancel{2}} = n^2.$

$1+3+5 = 9 = 3^2$

$1+3+5+7+9+11+13+15 = 64 = 8^2.$

n°15.



2) a) Chaque mètre supplémentaire coûte 50 € de plus que le précédent.
Donc (u_n) est une suite arithmétique de raison 50 et de 1er terme : $u_1 = 1000$.

$u_n = u_0 + nr.$

$u_n = u_p + (n-p)r.$

$u_n = u_1 + (n-1) \times 50.$

$u_n = 1000 + 50n - 50.$

$\boxed{u_n = 950 + 50n.}$

$\left(u_3 = 950 + 50 \times 3 = 950 + 150 = 1100. \right).$

b) Combien coûtent 2 m :

$u_1 + u_2$

3 m : $u_1 + u_2 + u_3.$

n m $= u_1 + u_2 + u_3 + \dots + u_n. = 519\,750.$

$$\frac{n \times (1000 + 950 + 50n)}{2} = 519\,750.$$

$$n(1950+50n) = 1\,039\,500.$$

$$1950n + 50n^2 = 1\,039\,500.$$

$$50n^2 + 1950n - 1\,039\,500 = 0.$$

$$\frac{50n^2+1950n-1\,039\,500}{50} = \frac{0}{50}.$$

$$\boxed{n^2 + 39n - 20790 = 0}$$