Séance du 02/10/19.

**TD sur Continuité.**

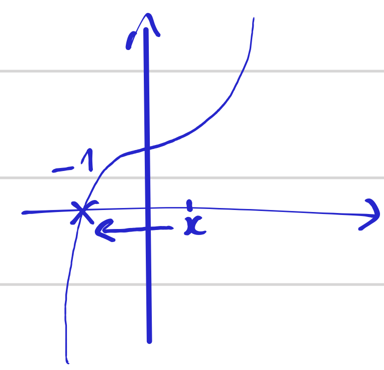
n° 5 suite :

1) d) D'après le tableau de variation, on obtient le signe de $u(x)$ :

| $x$ | $-\infty$ $\qquad$ $\alpha$ $\qquad$ $+\infty$ |
|---|---|
| $u(x)$ | $-$ $\quad$ 0 $\quad$ $+$ |

2) a) Etudions la limite de cette fonction en $-1$ et $x > -1$.
en $-1^+$

$\lim\limits_{\substack{x \to -1 \\ x > -1}} 1 - x = 1 + 1 = 2.$ $\qquad$ $\lim\limits_{\substack{x \to -1 \\ x > -1}} 1 + x^3 = 0^+.$




Par quotient des limites, $\lim\limits_{\substack{x \to -1 \\ x > -1}} f(x) = +\infty.$

$$\frac{-x}{x^3} = \frac{-\cancel{x}}{x^2 \times \cancel{x}} = \frac{-1}{x^2}$$

Etudions la limite de $f(x)$ en $+\infty$.

$$\lim_{x \to +\infty} \frac{1-x}{1+x^3} = \lim_{x \to +\infty} \frac{-x}{x^3} = \lim_{x \to +\infty} \frac{-1}{x^2} = 0$$

Autre méthode : $\dfrac{1-x}{1+x^3} = \dfrac{x\left(\frac{1}{x} - 1\right)}{x^3\left(\frac{1}{x^3} + 1\right)} = \dfrac{\frac{1}{x} - 1}{x^2\left(\frac{1}{x^3} + 1\right)}$

b) $f(x) = \dfrac{u(x)}{v(x)}$ , $f'(x) = \dfrac{u'(x) \times v(x) - v'(x) \times u(x)}{v^2(x)}$.

$f(x) = \dfrac{1-x}{1+x^3}$ $\quad u(x) = 1-x \quad v(x) = 1+x^3$

$u'(x) = -1 \quad v'(x) = 3x^2$

$$f'(x) = \frac{-1 \times (1+x^3) - 3x^2(1-x)}{(1+x^3)^2} = \frac{-1-x^3-3x^2+3x^3}{(1+x^3)^2}$$

$$f'(x) = \frac{2x^3-3x^2-1}{(1+x^3)^2} = \frac{u(x)}{(1+x^3)^2}.$$

c) On dresse le tableau de variat° de $f$ à l'aide du tableau de signe de $f'(x)$.

| $x$ | $-1$ | | $\alpha$ | | $+\infty$ |
|---|---|---|---|---|---|
| $u(x)$ | | $-$ | $0$ | $+$ | |
| $(1+x^3)^2$ | | $+$ | | $+$ | |
| $f'(x)$ | | $-$ | $0$ | $+$ | |
| $f(x)$ | $+\infty$ | $\searrow$ | $f(\alpha)$ | $\nearrow$ | $0$ |

d) $f(\alpha) = \dfrac{1-\alpha}{1+\alpha^3}$ $\qquad u(\alpha) = 0$

$2\alpha^3 - 3\alpha^2 - 1 = 0$.

$f(\alpha) = \dfrac{1-\alpha}{1+\frac{3\alpha^2+1}{2}}$. $\qquad \alpha^3 = \dfrac{3x^2+1}{2}$

$$f(\alpha) = \frac{1-\alpha}{\frac{2+3\alpha^2+1}{2}} = \frac{1-\alpha}{\frac{3\alpha^2+3}{2}} = \frac{2(1-\alpha)}{3(\alpha^2+1)}$$

3) $g(x) = x(x-1)$ $\qquad h(x) = \frac{1}{2}\left(x + \frac{1}{x}\right)$.

a) La calculatrice nous renseigne que :

Sur $]-\infty; 0[$, $\mathcal{C}_g$ est au-dessus de $\mathcal{C}_h$.

Sur $]0; 1{,}68[$, $\mathcal{C}_g$ est en-dessous de $\mathcal{C}_h$.

Sur $[1{,}68; +\infty$, $\mathcal{C}_g$ est au-dessus de $\mathcal{C}_h$.

b) $g(x) - h(x) = x(x-1) - \frac{1}{2}\left(x + \frac{1}{x}\right)$.

$$= \frac{x^2}{1} - \frac{x}{1} - \frac{x}{2} - \frac{1}{2x}$$

$$= \frac{x^2 \times 2x}{1 \times 2x} - \frac{x \times 2x}{1 \times 2x} - \frac{x \times x}{2 \times x} - \frac{1}{2x}.$$

$$= \frac{2x^3 - 2x^2 - x^2 + 1}{2x} = \frac{2x^3 - 3x^2 - 1}{2x}$$

$$= \frac{u(x)}{2x}.$$

c)

| $x$ | $-\infty$ | $0$ | $\alpha$ | $+\infty$ |
|---|---|---|---|---|
| $u(x)$ | $-$ | $-$ | $0$ | $+$ |
| $2x$ | $-$ | $0$ $+$ | | $+$ |
| $g(x) - h(x)$ | $+$ | $\|$ $-$ | | $+$ |
| Pos° relatives | $\mathcal{C}_g$ au dessus de $\mathcal{C}_h$ | $\mathcal{C}_g$ en dess $\mathcal{C}_h$ | | $\mathcal{C}_g$ au dessus de $\mathcal{C}_h$ |

n°7 : 6) 7) 9) 2) 3)

2) $f(x) = \dfrac{1-2x}{x-2}$ $\qquad \dfrac{u(x)}{v(x)} \rightarrow \dfrac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$

$u(x) = 1-2x. \qquad v(x) = x-2 \qquad ax+b \rightarrow a.$

$u'(x) = -2 \qquad v'(x) = 1$

$$f'(x) = \frac{-2(x-2) - 1(1-2x)}{(x-2)^2} = \frac{-\cancel{2x}+4-1+\cancel{2x}}{(x-2)^2} = \frac{3}{(x-2)^2}$$

3) $f(x) = x - 6 + \dfrac{9}{x-1} = x - 6 + 9 \times \dfrac{1}{x-1}.$ $\qquad \left(\dfrac{1}{u(x)}\right)' = \dfrac{-u'(x)}{u^2(x)}$

$$f'(x) = 1 + 9 \times \frac{(-1)}{(x-1)^2} = 1 - \frac{9}{(x-1)^2}.$$

$$f'(x) = \frac{(x-1)^2 - 3^2}{(x-1)^2} = \frac{(x-1+3)(x-1-3)}{(x-1)^2}$$

$$= \frac{(x+2)(x-4)}{(x-1)^2}.$$

6) $f(x) = \left(\dfrac{x+1}{x+2}\right)^3$ $\qquad (u(x)^2)' = 2u'(x)\,u(x)$

$$\left((u(x))^n\right)' = n\,u'(x)(u(x))^{n-1}.$$

$$f'(x) = 3 \times \left(\frac{1 \times (x+2) - 1(x+1)}{(x+2)^2}\right) \times \left(\frac{x+1}{x+2}\right)^2$$

$$f'(x) = 3 \times \frac{1}{(x+2)^2} \left(\frac{x+1}{x+2}\right)^2.$$

7) $f(x) = \cos(2x).$ $\quad$ $p(x) = g(f(x))$

$f'(x) = -2 \times \sin(2x).$ $\quad$ $p'(x) = f'(x) \times g'(f(x)).$

9) $f(x) = \sqrt{\dfrac{x+1}{2-x}}.$

Ex: $f(x) = 2x+3.$

$g(x) = x^2$

$$\left(\sqrt{u(x)}\right)' = u'(x) \times \frac{1}{2\sqrt{u(x)}}$$

$$= \frac{u'(x)}{2\sqrt{u(x)}}.$$

$p(x) = g(f(x)) = (f(x))^2 = (2x+3)^2.$

$p(x) = (2x+3)^2.$

$p'(x) = \underset{n}{2} \times \underset{u'}{2} \times \underset{u^{n-1}}{(2x+3)}.$

$$f'(x) = \frac{\dfrac{1 \times (2-x) - (-1) \times (x+1)}{(2-x)^2}}{2\sqrt{\dfrac{x+1}{2-x}}} = \frac{\dfrac{3}{(2-x)^2}}{2\sqrt{\dfrac{x+1}{2-x}}}.$$

**Exercice n°8 : (2).**

2) $f(x) = \dfrac{x}{x^2+1}$ $\quad a = 2$

Eq° de la tangente à $\mathcal{C}_f$ en 2 :

$y = f'(2)(x-2) + f(2)$

$$f'(x) = \frac{1 \times (x^2+1) - 2x \times (x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$$

$$f'(2) = \frac{-2^2+1}{(2^2+1)^2} = \frac{-4+1}{25} = \frac{-3}{25}$$

$$f(2) = \frac{2}{2^2+1} = \frac{2}{5}$$

$$y = f'(2)(x-2) + f(2)$$

$$y = -\frac{3}{25}(x-2) + \frac{2}{5}$$

$$y = -\frac{3}{25}x + \frac{6}{25} + \frac{2}{5}$$

$$y = -\frac{3}{25}x + \frac{6}{25} + \frac{10}{25}$$

$$y = -\frac{3}{25}x + \frac{16}{25}$$