24/12/19.

Fonction Exponentielle

Equations différentielle ?

$2x + 3 = 0.$

$2x^2 - 3x + 2 = 0.$

Ex : $y' = 2x$

$$\sum \vec{F}_{ext} = m \times v'.$$

$\lambda v = m v'.$ $\lambda = m.$

$\cancel{m} v = \cancel{m} v'$

$v' = v.$

$$\begin{cases} f'(x) = f(x). \\ f(0) = 1. \end{cases} \longrightarrow exp(x)$$

$f(x) = x^2$

$f(x) = exp(x)$ $exp(0) = 1.$

$f'(x) = exp(x).$

Démontrons que $exp(a+b) = exp(a) \times exp(b).$

Soit $j(x) = \dfrac{exp(x+a)}{exp(a)}$ où a est un réel fixé.

$$j(x) = \underbrace{\frac{1}{\exp(a)}}_{K \in \mathbb{R}} \times \underbrace{\exp(x+a)}_{f(x)}.$$

$$j'(x) = \frac{1}{\exp(a)} \times \exp(x+a). = \frac{\exp(x+a)}{\exp(a)} = j(x).$$

$$j(x) = \frac{\exp(x+a)}{\exp(a)}$$

$$j(0) = \frac{\exp(0+a)}{\exp(a)} = 1.$$

Or exp est unique

donc $j(x) = \exp(x).$

$$\frac{\exp(x+a)}{\exp(a)} = \exp(x)$$

$$\exp(x+a) = \exp(x) \times \exp(a).$$

$*\ \exp(-a) = \dfrac{1}{\exp(a)}$

$\exp(a) \times \exp(b) = \exp(a+b).$

$$\exp(-a) \times \exp(a) = \exp(-a+a) = \exp(0) = 1.$$

$$\exp(-a) \times \exp(a) = 1.$$

$$\exp(-a) = \frac{1}{\exp(a)}. \qquad \exp(-2) = \frac{1}{\exp(2)}.$$

$$a^{-1} = \frac{1}{a^1}$$

$$a^{-n} = \frac{1}{a^n}.$$

$$10^{-3} = 0{,}001.$$

$$e^x.$$

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0{,}001.$$

$$\exp(a+b) = \exp(a) \times \exp(b).$$

$*$ $\exp(a-b) = \frac{\exp(a)}{\exp(b)}$.

$$\exp(a-b) = \exp(a+(-b))$$
$$= \exp(a) \times \exp(-b)$$
$$= \exp(a) \times \frac{1}{\exp(b)} = \frac{\exp(a)}{\exp(b)}.$$

$*$ $\exp(na) = (\exp(a))^n$.

$$\exp(\underbrace{a+a+a+\dots+a}_{\text{"}na\text{"}}) = \overbrace{\exp(a) \times \exp(a) \times \exp(a) \times \dots \times \exp(a)}^{n}$$
$$= (\exp(a))^n.$$

$\forall x \in \mathbb{R}$ $e^x \neq 0$.

On suppose qu'il existe $x \in \mathbb{R}$ t.q $e^x = 0$.

Or $\exp(-x) \times \exp(x) = 1$.

$$e^{-x} \times e^x = 1.$$
$$e^{-x} \times 0 = 1.$$
$$0 = 1.$$ Absurde.

$\forall x \in \mathbb{R}$ $e^x \neq 0$

$\forall x \in \mathbb{R} \quad e^{x} > 0.$

$$\boxed{\left(e^{nx}\right) = \left(e^{x}\right)^{n}.}$$

$$e^{x} = e^{\frac{x}{2}\times 2} = \left(e^{\frac{x}{2}}\right)^{2} \geq 0.$$

Or $\forall x \in \mathbb{R} \quad e^{x} \neq 0$ donc $e^{x} > 0.$

alors $\Longrightarrow$ $\qquad x^2 = 4$

équivalent $\Longleftrightarrow$ $\qquad \cancel{\Longleftrightarrow} \; x = 2$

Si $x = 2$ alors $x^2 = 4.$

Si $x^2 = 4$ alors $x = 2$ ou $-2.$

$$\left(e^{a}\right)^{n} = 1 \times \left(e^{a}\right)^{n}.$$

$$u_0 \times q^{n}.$$

$$\left(e^{u(x)}\right)' = u'(x)\, e^{u(x)}.$$

Ex: $f(x) = e^{2x+2}$ $\qquad f'(x) = 2\, e^{2x+2}.$

$$\left(e^{x+a}\right)' = e^{x+a}.$$

Exercice 1

Simplifier les écritures suivantes :

1) $(e^x)^3 e^{-2x}$ 2) $\dfrac{e^{x-1}}{e^{x+2}}$ 3) $\dfrac{e^x + e^{-x}}{e^x}$ 4) $e^{-x}e^2$

5) $\dfrac{e^{3x}}{(e^{-x})^2 \times e^x}$ 6) $\dfrac{e^x e^y}{e^{x-y}}$ 7) $\dfrac{(e^{-3x})^2 e^{5x}}{e^{-x}}$

1) $(e^x)^3 e^{-2x} = e^{3x} \times e^{-2x} = e^{3x-2x} = e^x.$

2) $\dfrac{e^{x-1}}{e^{x+2}} = e^{x-1-(x+2)} = e^{\cancel{x}-1-\cancel{x}-2} = e^{-3} = \dfrac{1}{e^3}.$ $\dfrac{e^a}{e^b} = e^{a-b}.$

3) $\dfrac{e^x + e^{-x}}{e^x} = \dfrac{e^x}{e^x} + \dfrac{e^{-x}}{e^x} = 1 + e^{-x-x} = 1 + e^{-2x}$

4) $e^{-x} \times e^2 = e^{2-x}.$

5) $\dfrac{e^{3x}}{(e^{-x})^2 \times e^x} = \dfrac{e^{3x}}{e^{-2x} \times e^x} = \dfrac{e^{3x}}{e^{-2x+x}} = \dfrac{e^{3x}}{e^{-x}} = e^{3x-(-x)} = e^{4x}.$

6) $\dfrac{e^x e^y}{e^{x-y}} = \dfrac{e^{x+y}}{e^{x-y}} = e^{\cancel{x}+y-\cancel{x}+y} = e^{2y}.$

7) $$\frac{\left(e^{-3x}\right)^2 e^{5x}}{e^{-x}} = \frac{e^{-6x} \times e^{5x}}{e^{-x}} = \frac{e^{-x}}{e^{-x}} = 1.$$