26/09/22.

P-G- Spémaths: Suites.

$\mathbb{N}$

$n \longmapsto u_n$

Explicite:

$u_n = f(n)$.

Récurrente:

$u_{n+1} = f(u_n)$.

Généralités sur les suites

Exercice 1

Pour les suites suivantes, trouver la fonction f associée à la suite définie par la relation de récurrence $u_{n+1} = f(u_n)$ et calculer les termes de u_1 à u_4

a) $\begin{cases} u_0 = \frac{5}{1} \\ u_{n+1} = \dfrac{2u_n}{u_n+1} \end{cases}$ b) $\begin{cases} u_0 = -1 \\ u_{n+1} = (u_n+1)^2 \end{cases}$ c) $\begin{cases} u_0 = 2 \\ u_{n+1} = \dfrac{u_n - 1}{u_n} \end{cases}$ d) $\begin{cases} u_0 = 1 \\ u_{n+1} = \sqrt{u_n+1} \end{cases}$

a) $u_1 = \dfrac{2u_0}{u_0+1} = \dfrac{2\times 5}{5+1} = \dfrac{10}{6} = \dfrac{5}{3}$.

$u_2 = \dfrac{2u_1}{u_1+1} = \dfrac{2\times\frac{5}{3}}{\frac{5}{3}+1} = \dfrac{\frac{10}{3}}{\frac{5}{3}+\frac{3}{3}} = \dfrac{\frac{10}{3}}{\frac{8}{3}} = \dfrac{10}{\cancel{3}} \times \dfrac{\cancel{3}}{8} = \dfrac{10}{8} = \dfrac{5}{4}$.

$u_3 = \dfrac{2u_2}{u_2+1} = \dfrac{2\times\frac{5}{4}}{\frac{5}{4}+1} = \dfrac{\frac{10}{4}}{\frac{5}{4}+\frac{4}{4}} = \dfrac{\frac{10}{4}}{\frac{9}{4}} = \dfrac{10}{\cancel{4}} \times \dfrac{\cancel{4}}{9} = \dfrac{10}{9}$.

$u_4 = \dfrac{2u_3}{u_3+1} = \dfrac{2\times\frac{10}{9}}{\frac{10}{9}+1} = \dfrac{\frac{20}{9}}{\frac{10}{9}+\frac{9}{9}} = \dfrac{\frac{20}{9}}{\frac{19}{9}} = \dfrac{20}{\cancel{9}} \times \dfrac{\cancel{9}}{19}$.

d) $\begin{cases} u_0 = 1 \\ u_{n+1} = \sqrt{u_n+1} \end{cases}$

$u_1 = \sqrt{u_0+1} = \sqrt{1+1} = \sqrt{2}$.

$u_2 = \sqrt{u_1+1} = \sqrt{\sqrt{2}+1}$.

$$U_3 = \sqrt{U_2 + 1} = \sqrt{\sqrt{\sqrt{2}+1} + 1}$$

$$U_4 = \sqrt{U_3 + 1} = \sqrt{\sqrt{\sqrt{\sqrt{2}+1} + 1} + 1}$$

Soit $n \in \mathbb{N}$. $U_n = n^2 + 2n + 2$.

$U_n = f(n)$

$f(x) = x^2 + 2x + 2$.

$f(2) = 2^2 + 2 \times 2 + 2$.

$f(9) = 9^2 + 2 \, 9 + 2$.

$f(n) = n^2 + 2n + 2 = U_n$.

Application : 1 $\quad U_n = \dfrac{1}{n^2+4}$ $\quad U_0 = \dfrac{1}{0^2+4} = \dfrac{1}{4}$.

$U_1 = \dfrac{1}{1^2+4} = \dfrac{1}{5}$.

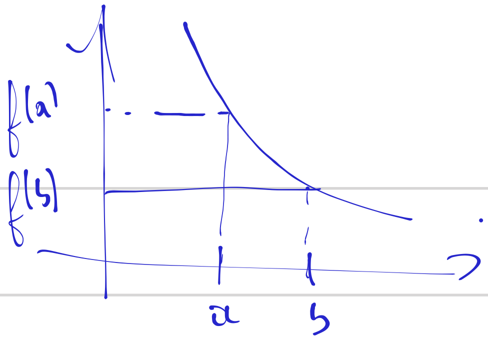
$U_2 = \dfrac{1}{2^2+4} = \dfrac{1}{8}$.

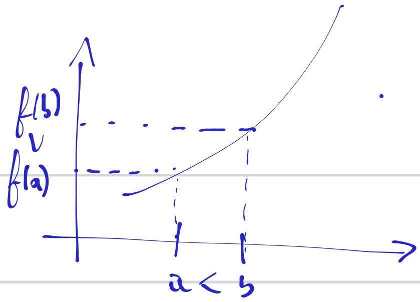
Application 2 : $\begin{cases} U_{n+1} = U_n + 2n + 1. & n \in \mathbb{N} \\ U_0 = 0. \end{cases}$

$U_1 = U_0 + 2 \times 0 + 1 = 1$.

$U_2 = U_1 + 2 \times 1 + 1 = 1 + 2 + 1 = 4$.

$U_3 = U_2 + 2 \times 2 + 1 = 4 + 4 + 1 = 9$.





$u_0 = 1$

$u_1 = 3$

$u_2 = 8$

$u_3 = 2.$

$\forall n \in \mathbb{N} \quad u_{n+1} \geq u_n$

Applicat° 3 : $n \in \mathbb{N} \quad u_n = n^2.$ $\quad 8 ; 7 ; 6 ; 5 ; 4$

$u_{n+1} - u_n = (n+1)^2 - n^2$ $\quad (a+b)^2 = a^2 + 2ab + b^2.$

$= \cancel{n^2} + 2n + 1 - \cancel{n^2}$

$= 2n+1.$ Or $n \geq 0$ car $n \in \mathbb{N}.$

$\times 2$

$2n \geq 0$

$+1$

$2n+1 \geq 1 > 0.$

$\longrightarrow u_{n+1} - u_n > 0.$ $\quad + u_n.$

$u_{n+1} - \cancel{u_n} + \cancel{u_n} > u_n.$

$\boxed{u_{n+1} > u_n.}$

$\begin{cases} u_{n+1} = \frac{1}{2} u_n \\ u_0 = 10 \end{cases}$

$u_1 = \frac{1}{2} u_0 = \frac{1}{2} \times 10 = 5.$

$\frac{a^b}{a^c} = a^{b-c}.$

App 4 : $n \in \mathbb{N}$ $u_n = 2^n$. $\frac{u_{n+1}}{u_n} = \frac{2^{n+1}}{2^n} = 2^{n+1-n} = 2^1 = 2 > 1$.

$\forall n \in \mathbb{N}\ u_n > 0$.

Car $u_n = \underbrace{2 \times 2 \times 2 \times \dots \times 2}_{n \text{ fois } 2}$.

Donc (u_n) est croissante.

$a^b \times a^c = a^{b+c}$

Méthode 1

$u_{n+1} - u_n = 2^{n+1} - 2^n = 2 \times 2^n - 2^n \times 1$.

$= 2^n(2-1)$

$= 2^n \times 1 = 2^n > 0$.

Exercice 3

Pour les exercices suivants, étudier le sens de variation de la suite (u_n).

a) $u_n = \dfrac{3n-2}{n+1}$

b) $u_n = \dfrac{2^{3n}}{3^{2n}}$

c) $u_n = (n-5)^2, \quad n \geqslant 5$

d) $u_0 = 2$ et $\forall n \in \mathbb{N},\ u_{n+1} = u_n - n$

e) $\forall n \in \mathbb{N}^*, u_n = \dfrac{2^n}{n}$

Soit $n \in \mathbb{N}$

a) $u_{n+1} - u_n = \dfrac{3(n+1)-2}{n+1+1} - \dfrac{3n-2}{n+1}$

$= \dfrac{3n+3-2}{n+2} - \dfrac{3n-2}{n+1}$

$\frac{1}{3} + \frac{1}{4}$

$= \dfrac{(n+1)(3n+1)}{(n+1) \times (n+2)} - \dfrac{(3n-2) \times (n+2)}{(n+1) \times (n+2)}$

$= \dfrac{3n^2+n+3n+1 - (3n^2+6n-2n-4)}{(n+1)(n+2)}$

$$= \frac{\cancel{3n^2} + \cancel{4n} + 1 - \cancel{3n^2} - \cancel{4n} + 4}{(n+1)(n+2)} = \frac{5 \; (+)}{(n+1)(n+2)}$$

$$(+) \quad (+)$$

$u_{n+1} - u_n$ est positif donc (u_n) est croissante.

b) $u_n = \frac{2^{3n}}{3^{2n}}$ Soit $n \in \mathbb{N}$. $$\frac{u_{n+1}}{u_n} = \frac{\frac{2^{3(n+1)}}{3^{2(n+1)}}}{\frac{2^{3n}}{3^{2n}}}$$

$$\frac{a^c}{b^c} = \left(\frac{a}{b}\right)^c$$

$$= \frac{\left(\frac{2^3}{3^2}\right)^{n+1}}{\left(\frac{2^3}{3^2}\right)^{n}} = \left(\frac{2^3}{3^2}\right)^{n+1-n} = \frac{2^3}{3^2} = \frac{8}{9} < 1$$

(u_n) est donc décroissante.

Pour le 03/09/20 : Finir le 3 en entier et représenter graphiquement la suite (u_n) définie par :

$$\forall n \in \mathbb{N} : \begin{cases} u_{n+1} = 2u_n + 3. \\ u_0 = 1. \end{cases}$$

u_0 ; u_1 ; u_2 et u_3.