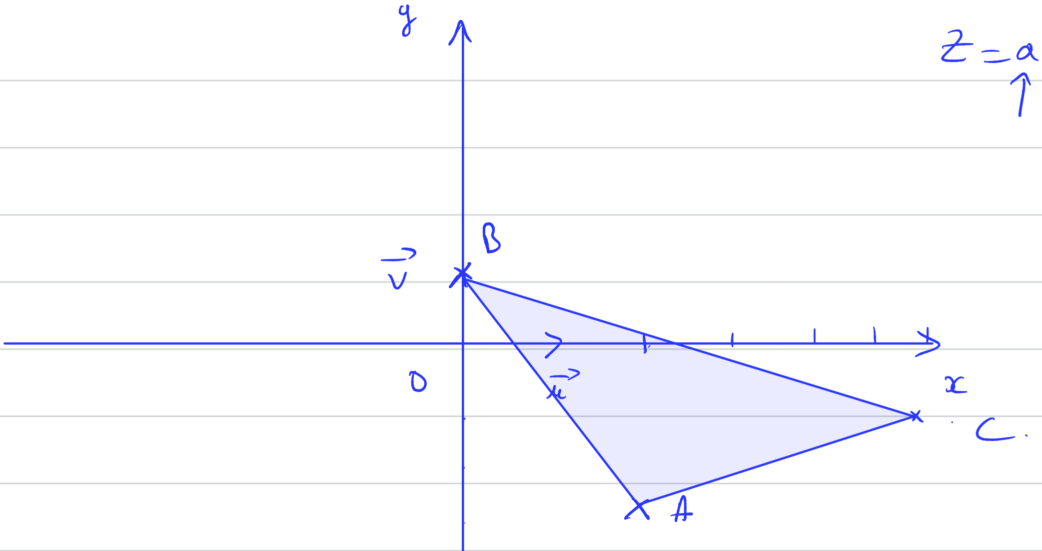
n°37 :

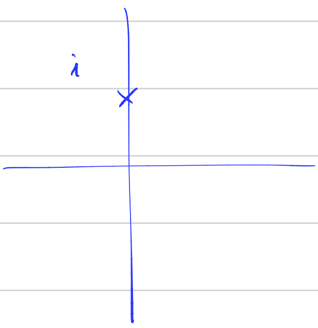



$z = a + ib$

1) $\dfrac{z_B - z_A}{z_C - z_A} = \dfrac{i - (2-3i)}{6 - i - (2-3i)}$

$= \dfrac{i - 2 + 3i}{6 - i - 2 + 3i} = \dfrac{(-2+4i)(4-2i)}{(4+2i)(4-2i)} = \dfrac{-8 + 4i + 16i - 8i^2}{4^2 - (2i)^2} = \dfrac{20i - 8 + 8}{16 + 4}$

$= \dfrac{\cancel{20}i}{\cancel{20}} = i.$




2) $\operatorname{Arg}\left(\dfrac{z_B - z_A}{z_C - z_A}\right) = (\vec{AC} ; \vec{AB}) = \operatorname{Arg}(i) = \dfrac{\pi}{2}.$

Donc ABC est rectangle en A

Or $AB = |z_B - z_A| = |i - (2-3i)| = |i - 2 + 3i|$

$= |-2 + 4i|$

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16} = \sqrt{20}$

$= \sqrt{4 \times 5}$

$= 2\sqrt{5}.$

$AC = |z_C - z_A|$ $\quad |a+ib| = \sqrt{a^2+b^2}.$

$= |6 - i - 2 + 3i| = |4 - 2i|$

$= \sqrt{4^2 + (-2)^2}$

$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}.$

Le triangle ABC est donc rectangle isocèle.

PB :

$$z' = f(z) = \frac{i(z-2+3i)}{z-i}$$

1) $z_D = 1-i$.

$$z_{D'} = f(z_D) = \frac{i(1-i-2+3i)}{1-i-i}$$

$$z_{D}' = \frac{i(-1+2i)}{1-2i} = \frac{(-i-2)(1+2i)}{(1-2i)(1+2i)} = \frac{-i+2-2-4i}{1^2+2^2}$$

$$z_D' = \frac{-5i}{5} = -i.$$

2)a) $z' = 2i$ $\qquad z \neq i$

$$\frac{i(z-2+3i)}{z-i} = 2i.$$

$$\cancel{i}(z-2+3i) = 2\cancel{i}(z-i)$$

$$z-2+3i = 2z-2i$$

$$-2+5i = z_E.$$

b) $z_{\vec{AE}} = z_E - z_A = -2+5i-(2-3i)$

$= -2+5i-2+3i$

$= -4+8i.$

$z_{\vec{AB}} = z_B - z_A = -2+4i$

$z_{\vec{AE}} = 2 \times z_{\vec{AB}}.$

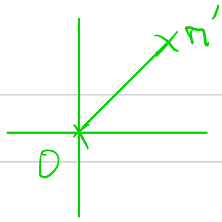
$\vec{AE} = 2\vec{AB}.$

Donc A, E et B sont alignés. $E \in (AB)$

3) $z'$ est l'affixe de $M'$

$z$ " " " $M$

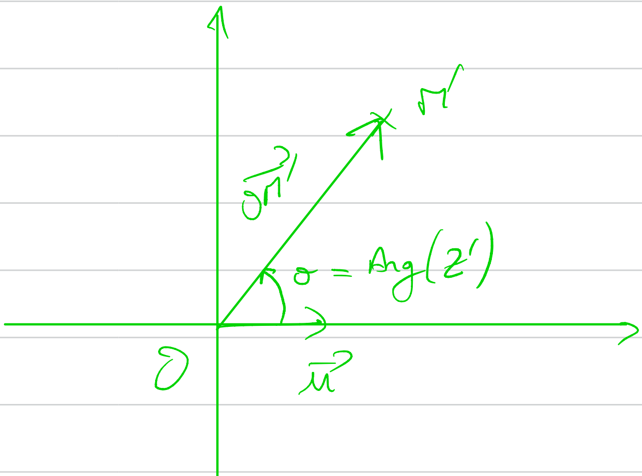
$$z' = \frac{i(z-2+3i)}{z-i}$$




$$OM' = |z'| = \left|\frac{i(z-(2-3i))}{z-i}\right|$$

$$OM' = \left|\frac{i(z-z_A)}{z-z_B}\right| = |i| \times \frac{|z_M - z_A|}{|z_M - z_B|}$$

$$\boxed{OM' = \frac{AM}{BM}}$$



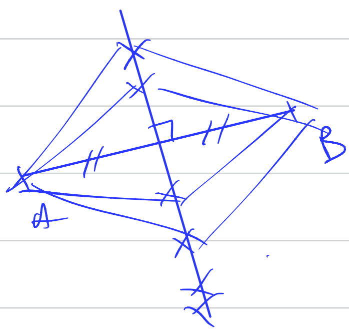

4) Soit $M$ un point distinct de $A$ et de $B$.

$$(\vec{u} ; \vec{OM'}) = \text{Arg}(z')$$

$$= \text{Arg}\left(\frac{i(z-z_A)}{z-z_B}\right)$$

$(\vec{u} ; \vec{v}) = -(\vec{v} ; \vec{u})$.

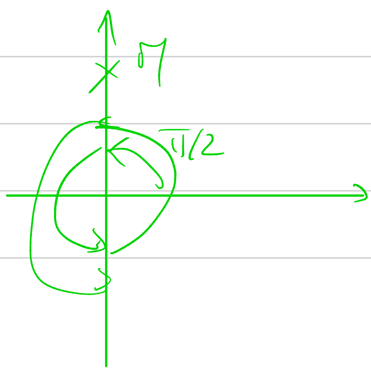
$$= \text{Arg}(i) + \text{Arg}\left(\frac{z-z_A}{z-z_B}\right) = \frac{\pi}{2} + (\vec{BM} ; \vec{AM}).$$




5) $M \in$ médiatrice de $[AB] \iff AM = BM$.

$$OM' = \frac{AM}{BM} \iff OM' = 1.$$

$$M' \in \mathcal{C}_{O;1}.$$



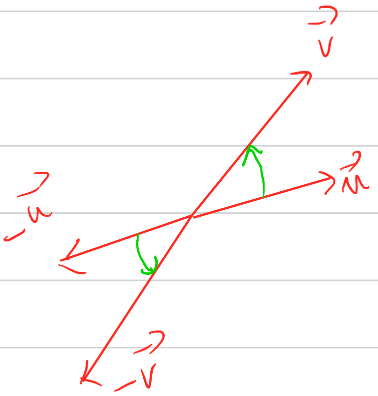

6) $(\vec{u} ; \vec{OM'}) = \frac{\pi}{2}\ [\pi] = \frac{\pi}{2} + k\pi$.

D'après 4 : $(\vec{u};\vec{OM'}) = (\vec{BM};\vec{AM}) + \frac{\pi}{2} + 2k\pi$

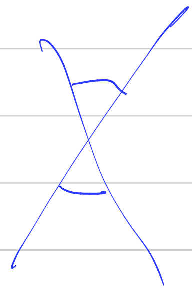
$$k\pi + \cancel{\frac{\pi}{2}} = (\vec{BM};\vec{AM}) + \cancel{\frac{\pi}{2}} + 2k\pi$$

$(\vec{m};\vec{v}) = (-\vec{u};-\vec{v})$.

$$(\vec{BM};\vec{AM}) = 0\ [\pi]$$
$$(\vec{MB};\vec{MA}) = 0\ [\pi].$$







Donc A, M et B st alignés. $M \in (AB)$

Exercice 1 : $z = a+ib$.

$z = |z|\, e^{i\theta}$ où $\theta = \mathrm{Arg}(z)$.

1°) $\left|\frac{7\sqrt{3}}{2} - \frac{7}{2}i\right| = \sqrt{\left(\frac{7\sqrt{3}}{2}\right)^2 + \left(\frac{7}{2}\right)^2} = \sqrt{\frac{49\times 3}{4} + \frac{49}{4}} = 7.$

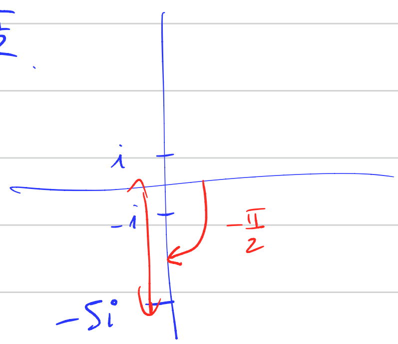
$\cos\theta = \frac{\frac{7\sqrt{3}}{2}}{7} = \frac{\sqrt{3}}{2}$

$\theta \equiv -\frac{\pi}{6}\ [2\pi].$

$\sin\theta = \frac{-\frac{7}{2}}{7} = -\frac{1}{2}$

$z = 7e^{-i\frac{\pi}{6}}.$

2°) $-5i = 5e^{-i\frac{\pi}{2}}.$




3°) $= \underline{-1\times e^{-i\frac{2\pi}{3}}}.$

$= e^{i\pi} \times$

$\cos(\theta)+i\sin(\theta) = e^{i\theta}$

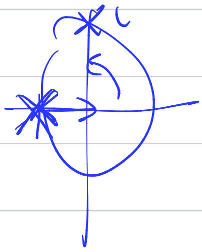
$-\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)$

$(a+b)^2 = a^2+2ab+b^2$

$(m+n)^2 = m^2+2mn+n^2$

$-e^{\frac{-2i\pi}{3}}$

$e^{i\pi} = \cos(\pi)+i\sin(\pi)$
$= -1$




$e^{i\pi}\times e^{-\frac{2i\pi}{3}}$

$e^{i\left(\pi-\frac{2\pi}{3}\right)} = \boxed{e^{i\pi/3}}$

$\frac{3}{2}+\frac{2}{3}$

9

6

4°) $-3ie^{2i\frac{\pi}{3}} = 3e^{i\pi}\times e^{i\pi/2}\times e^{2i\pi/3}$

$= 3e^{i\left(\pi+\frac{\pi}{2}+\frac{2\pi}{3}\right)} = 3e^{\frac{13\pi}{6}i}$

5°) $\frac{\left(e^{-5i\frac{\pi}{4}}\right)^2}{e^{-i\frac{\pi}{6}}} = \frac{e^{-\frac{5i\pi}{2}}}{e^{-i\frac{\pi}{6}}} = e^{i\left(-\frac{5\pi}{2}+\frac{\pi}{6}\right)}$

$= e^{-\frac{7\pi}{3}i}$

$z = \underbrace{|z|}_{>0}e^{i\theta}$

$2+3i$

$\sin(x) = \cos\left(\frac{\pi}{2}-x\right)$

$\cos(x) = \sin\left(\frac{\pi}{2}-x\right)$

n°2: 1) $\sin(\alpha)+i\cos(\alpha)$

$= \cos\left(\frac{\pi}{2}-\alpha\right)+i\sin\left(\frac{\pi}{2}-\alpha\right)$

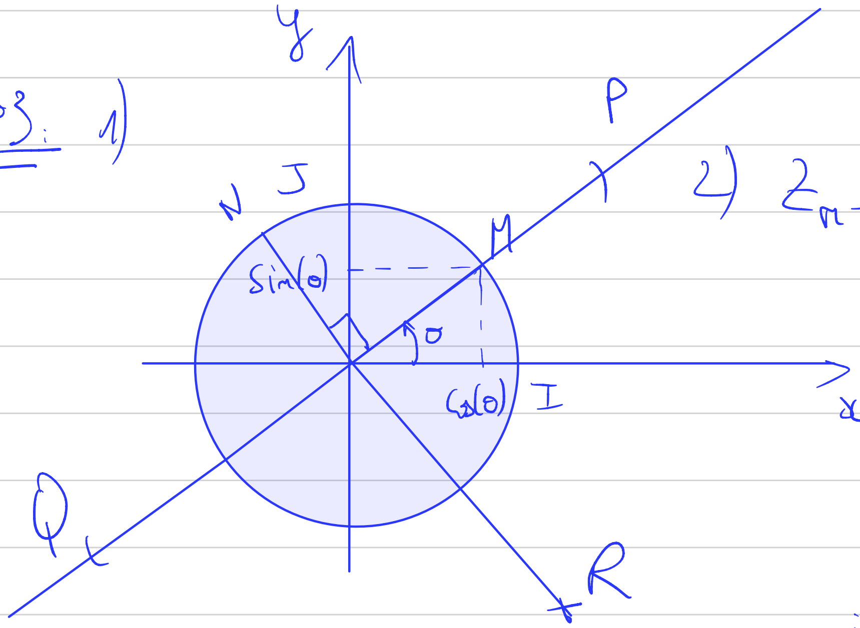
$= e^{i\left(\frac{\pi}{2}-\alpha\right)}$

2°) $-\cos(\alpha) - i\sin(\alpha) = -1(\cos(\alpha) + i\sin\alpha)$.

$= e^{i\pi} \; e^{i\alpha}$.

$= e^{i(\pi+\alpha)}$.

n°3 : 1)




2) $z_M = \cos(\theta) + i\sin(\theta) = e^{i\theta}$.

3) $z_N = ie^{i\theta} = e^{i\pi/2} \times e^{i\theta} = e^{i(\theta+\frac{\pi}{2})}$

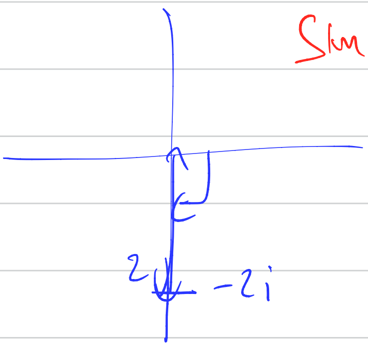
4)

5) $z_Q = -2e^{i\theta} = 2 \times e^{i(\theta+\pi)}$.

6) $z_R = -2i\, e^{i\theta}$

$= 2e^{-i\frac{\pi}{2}} e^{i\theta}$

$= 2e^{i(\theta-\frac{\pi}{2})}$.




$\sin(a+b) = \sin a \cos b + \sin b \cos a$.

$\sin(3\theta) = (\sin \theta + 2\theta)$.

$= \sin(\theta)\cos(2\theta) + \sin(2\theta)\cos(\theta)$.