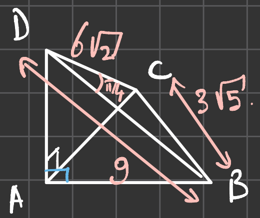
TG : Spé - maths - Géométrie dans l'espace

Samedi 22 janvier 2022

EXO 30 :

PARTIE B

$\sqrt{27} = \sqrt{9 \times 3}$
$= 3$



$\vec{u}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\|\vec{u}\| = \sqrt{x^2+y^2+z^2}$

2) $V(ABCD) = \frac{1}{3} \times \mathcal{A}(ABC) \times AD.$

$= \frac{1}{3} \times \frac{AB \times AC}{2} \times AD.$

$= \frac{1}{3} \times \frac{\sqrt{3^2+3^2+3^2} \times \sqrt{3^2+0^2+(-3)^2}}{2} \times \sqrt{(-3)^2+6^2+(-3)^2}$

$= \frac{1}{6} \times \sqrt{27} \times \sqrt{18} \times \sqrt{54}$

$= \frac{1}{6} \times 3\sqrt{3} \times 3\sqrt{2} \times 3\sqrt{6}.$

$= \frac{27}{6} \times \sqrt{36}$

$= \frac{27}{\cancel{6}} \times \cancel{6} = \boxed{27}.$

3. Calculons les coordonnées suivantes :

$B\begin{pmatrix} 6 \\ 1 \\ 5 \end{pmatrix}$ $D\begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$ $C\begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix}$.

$\vec{DB}\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$ $\vec{DC}\begin{pmatrix} 6 \\ -6 \\ 0 \end{pmatrix}$.

$\vec{BC}\begin{pmatrix} 0 \\ -3 \\ -6 \end{pmatrix}$ $\|\vec{BC}\| = BC = \sqrt{0^2+(-3)^2+(-6)^2}$
$= \sqrt{45}$
$= 3\sqrt{5}.$

D'où $\vec{DB} \cdot \vec{DC} = 6 \times 6 + (-3) \times (-6) + 6 \times 0 = 36 + 18 = 54.$

Or $\vec{DB} \cdot \vec{DC} = \|\vec{DB}\| \times \|\vec{DC}\| \times \cos(\widehat{BDC}) = \sqrt{6^2+(-3)^2+6^2} \times \sqrt{6^2+(-6)^2+0^2} \times \cos(\widehat{BDC}).$

$\sqrt{81} \times \sqrt{72} \times \cos(\widehat{BDC})$

$= 9 \times \sqrt{36 \times 2} \cos(\widehat{BDC})$

$= 9 \times 6 \times \sqrt{2} \cos(\widehat{BDC})$

$= 54\sqrt{2} \cos(\widehat{BDC})$

D'où : $54\sqrt{2}\cos(\widehat{BDC}) = 54$

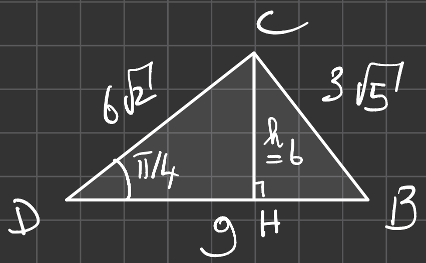
$\cos(\widehat{BDC}) = \frac{\cancel{54}}{\cancel{54}\sqrt{2}}$

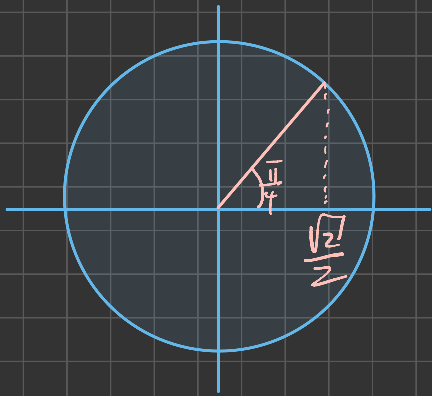
$\cos(\widehat{BDC}) = \frac{1}{\sqrt{2}}$

$\cos(\widehat{BDC}) = \frac{\sqrt{2}}{2}.$

$\widehat{BDC} = \text{Arcos}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$

4.a.





$$\sin(B\hat{D}C) = \frac{h}{DC}$$

$$h = DC \times \sin(B\hat{D}C)$$

$$h = 6\sqrt{2} \times \sin\left(\frac{\pi}{4}\right).$$

$$h = 6\sqrt{2} \times \frac{\sqrt{2}}{2}.$$

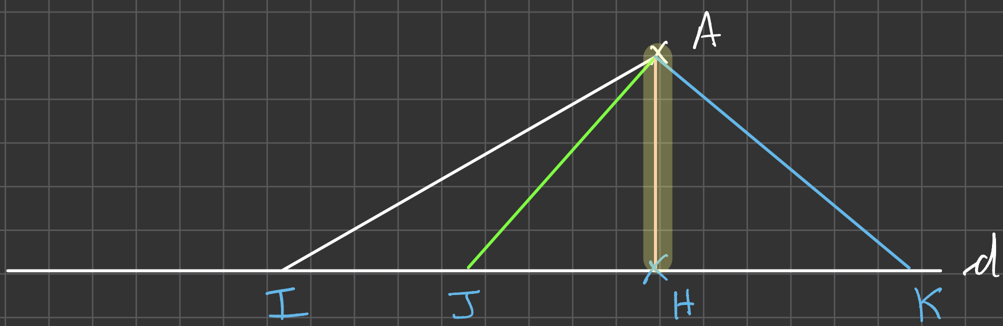
$$h = \frac{6 \times \cancel{2}}{\cancel{2}} = 6.$$

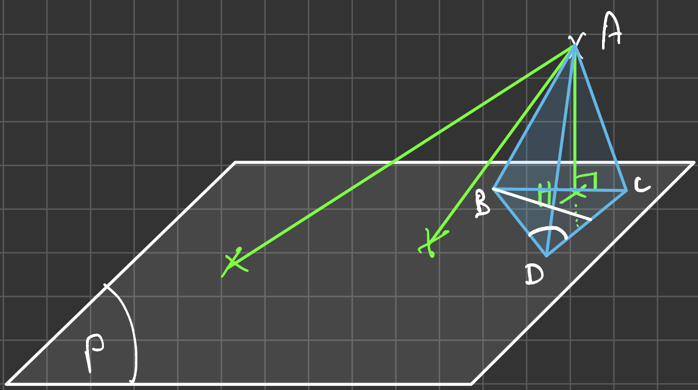
$$A(BDC) = \frac{DB \times h}{2}$$

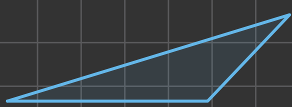
$$A(BDC) = \frac{9 \times 6}{2} = 9 \times 3$$

$$= 27.$$

4.b.







$$V(ABCD) = \frac{1}{3} \times A(BDC) \times h.$$

$$h = \frac{3V(ABCD) \; m^3}{A(BDC) \; m^2} = \frac{3 \times \cancel{27}}{\cancel{27}} = \boxed{3.}$$

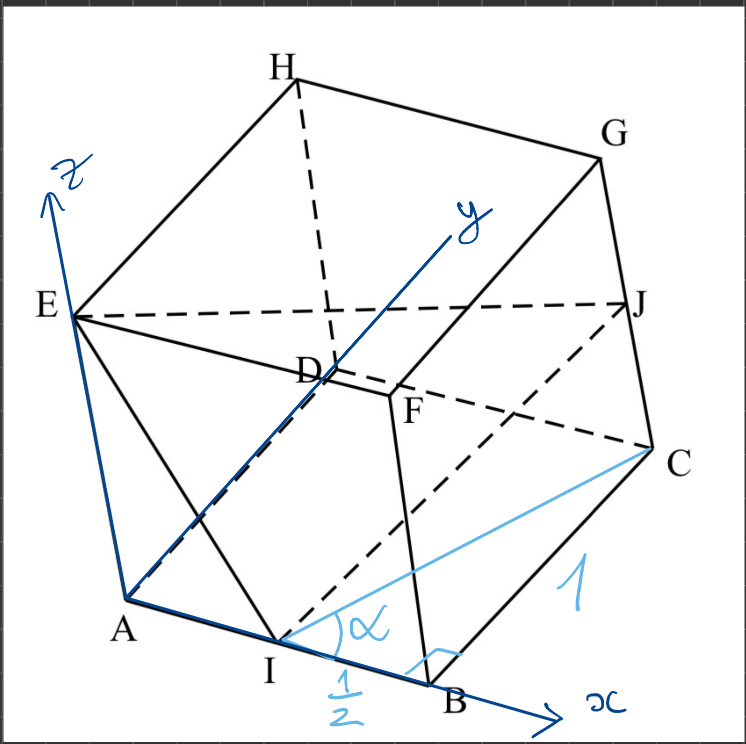
n°33 :

1) $\vec{AC}\cdot\vec{AI} = (\vec{AB}+\vec{BC})\cdot\vec{AI}$

$= \vec{AB}\cdot\vec{AI} + \underbrace{\vec{BC}\cdot\vec{AI}}_{0}$

$= 1 \times \frac{1}{2} = \frac{1}{2}$.

VRAI



2) $\vec{AI}\cdot\vec{AB} = \frac{1}{2}$ VRAI.

3) $\vec{AB}\cdot\vec{IJ} = \vec{AB}\cdot(\vec{IC}+\vec{CJ})$.

$= \vec{AB}\cdot\vec{IC} + \underbrace{\vec{AB}\cdot\vec{CJ}}_{=0}$.

$= \vec{AB}\cdot\vec{IC}$ VRAI.

4) $\vec{AB}\cdot\vec{IJ} = \vec{AB}\cdot\vec{IC}$

$= AB \times IC \times \cos\underbrace{(\vec{AB};\vec{IC})}_{\frac{\pi}{3}\,?}$

$\tan(\alpha) = \dfrac{1}{\frac{1}{2}} = 2$. $\alpha \neq \frac{\pi}{3}$ FAUX.

5. $\vec{IJ}\begin{pmatrix}\frac{1}{2}\\ 1\\ \frac{1}{2}\end{pmatrix}$ $2\vec{IJ}\begin{pmatrix}1\\ 2\\ 1\end{pmatrix}$. $I\begin{pmatrix}\frac{1}{2}\\ 0\\ 0\end{pmatrix}$ $J\begin{pmatrix}1\\ 1\\ \frac{1}{2}\end{pmatrix}$

$\vec{IJ}\begin{cases} x = \frac{1}{2} + t \\ y = 2t \\ z = t \end{cases}$ FAUX.

6. $\begin{cases} x = \frac{1}{2}t + 1 \\ y = t + 1 \\ z = \frac{1}{2}t + \frac{1}{2} \end{cases}$ $\vec{IJ}\begin{pmatrix}\frac{1}{2}\\ 1\\ \frac{1}{2}\end{pmatrix}$. VRAI.

7. FAUX.

8.