jeudi 17 août 2023.

1ère G - Spé maths.

Rappels :

① fonction du second degré :

$$f(x) = ax^2 + bx + c.$$

$$a \in \mathbb{R}^* \quad b \in \mathbb{R} \quad c \in \mathbb{R}$$

Ex : $f(x) = 2x^2 + 3x - 1.$

$$a = 2 \quad b = 3 \quad c = -1.$$

② Forme canonique : toutes les fonctions polynomes du second degré peuvent se mettre sous forme canonique.

$$f(x) = ax^2 + bx + c.$$

$$\Downarrow$$

$$f(x) = a(x - \alpha)^2 + \beta.$$ forme canonique

$$\frac{4}{8} = \frac{1}{2}$$

$$(m + p)^2 = m^2 + 2mp + p^2.$$

Démonstration : $f(x) = ax^2 + bx + c$ forme développée

$$= a \times \left(x^2 + \frac{b}{a} \times x\right) + c.$$

$$= a \times \left(x^2 + 2 \times x \times \frac{b}{2a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c.$$

$$= a \times \left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c.$$

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

$$= a \times \left(x + \frac{b}{2a}\right)^2 - a \times \left(\frac{b}{2a}\right)^2 + c.$$

$$= a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \cancel{a} \times \frac{b^2}{4\cancel{a} \times a} + c.$$

$$= a\left(x - \left(\frac{-b}{2a}\right)\right)^2 - \frac{b^2}{4a} + \frac{c_{\times 4a}}{1_{\times 4a}}$$

$$= a\left(x - \left(\frac{-b}{2a}\right)\right)^2 + \frac{-b^2 + 4ac}{4a}$$

$$= a(x - \alpha)^2 + \beta$$

$$\alpha = \frac{-b}{2a} \qquad \beta = \frac{-b^2 + 4ac}{4a}$$

Application Exo 1.

1) $f(x) = x^2 + 6x - 8$

$a = 1 \quad b = 6 \quad c = -8$

$$\alpha = \frac{-6}{2 \times 1} = -3$$

$$\beta = \frac{-6^2 + 4 \times 1 \times (-8)}{4 \times 1} = \frac{-36 - 32}{4}$$

$$\beta = \frac{-68}{4} = -17$$

Forme canonique :

$$f(x) = 1 \times (x + 3)^2 - 17$$

$$= x^2 + 6x - 8$$

5|6| 5) $3x^2 + 12x + 12$

Méth 1 : $a = 3 \qquad b = 12 \qquad c = 12$

$$\alpha = \frac{-b}{2a} = \frac{-12}{2 \times 3} = \frac{-12}{6} = -2$$

$$\beta = \frac{-b^2 + 4ac}{4a}$$

$$\beta = \frac{-12^2 + 4 \times 3 \times 12}{4 \times 3}$$

$$\beta = \frac{-144 + 144}{12} = 0$$

$f(x) = 3(x+2)^2$

Méth 2 : Trouver α et β.

$\alpha = \frac{-b}{2a} = \frac{-12}{2\times3} = -2.$

$\beta = f(\alpha) = 3\times(-2)^2 + 12\times(-2) + 12.$

$= 3\times4 - 24 + 12.$

$= 0.$

Méthode 3 : $f(x) = 3x^2 + 12x + 12.$

$= 3\left(x^2 + \frac{12}{3}x\right) + 12.$

$= 3(x^2 + 4x) + 12$

$= 3(x^2 + 2\times x\times 2 + 2^2 - 2^2) + 12.$

$= 3((x+2)^2 - 2^2) + 12.$

$= 3(x+2)^2 - 12 + 12$

$= 3(x+2)^2$

6) $f(x) = -x^2 + 7x - 10.$

$a = -1 \quad b = 7 \quad c = -10.$

$\alpha = \frac{-b}{2a} = \frac{-7}{2\times(-1)} = \frac{7}{2}$

$\beta = f(\alpha) = -\left(\frac{7}{2}\right)^2 + 7\times\frac{7}{2} - 10.$

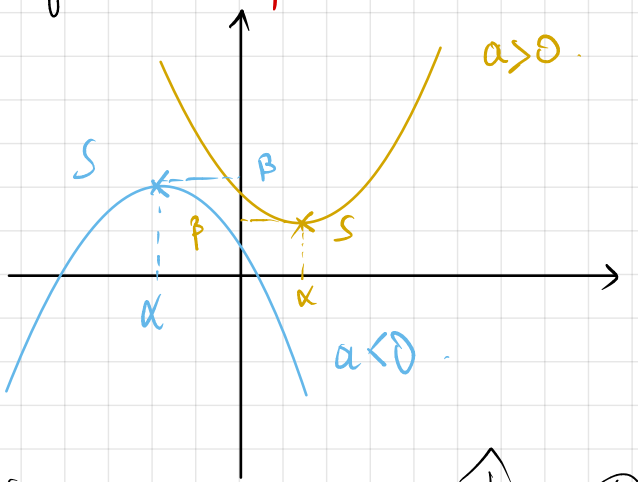
$= -\frac{49}{4} + \frac{49}{2} - 10 = -\frac{49}{4} + \frac{98}{4} - \frac{40}{4}$

$$= -\frac{89}{4} + \frac{98}{4} = +\frac{9}{4}.$$

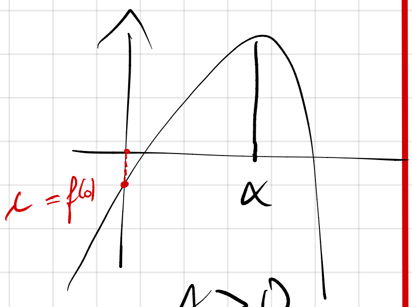
$$f(x) = -1\times\left(x - \frac{7}{2}\right)^2 + \frac{9}{4}.$$

(3) Représentation graphique des polynômes du second degré

Les graphes des polynômes du second degré sont des *paraboles*.



Exercice n°2 :



$a>0$.

$$\alpha = -\frac{b}{2a}$$

1) b)

2) a)

3) b) $f(x) = ax^2 + bx + c.$

$f(0) = a\times 0^2 + b\times 0 + c.$

$f(0) = c.$

4)

plus de bonnes notes
le soutien scolaire connecté

4 Equations du second degré:

$$f(x) = ax^2 + bx + c.$$

Déf: $f(x) = 0.$

$$ax^2 + bx + c = 0.$$

$$a(x-\alpha)^2 + \beta = 0.$$

$$a(x-\alpha)^2 = -\beta.$$

$$a(x-\alpha)^2 = -\left(\frac{-b^2+4ac}{4a}\right)$$

$$a(x-\alpha)^2 = \frac{b^2-4ac}{4a}$$

$$(x-\alpha)^2 = \frac{b^2-4ac}{4a^2} \quad \Delta$$

$$(x-\alpha)^2 = \frac{\Delta}{4a^2}$$

1er cas: Si $\Delta > 0$ alors:

$$(x-\alpha)^2 = \frac{\Delta}{4a^2}.$$

$$x-\alpha = \sqrt{\frac{\Delta}{4a^2}} \quad \text{ou} \quad x-\alpha = -\sqrt{\frac{\Delta}{4a^2}}.$$

$x^2 = 4 \quad x = 2$ et $x = -2.$

$$x^2 = -8.$$

$$x^2 = 8.$$

$$x = \sqrt{8} \quad \text{ou} \quad x = -\sqrt{8}.$$