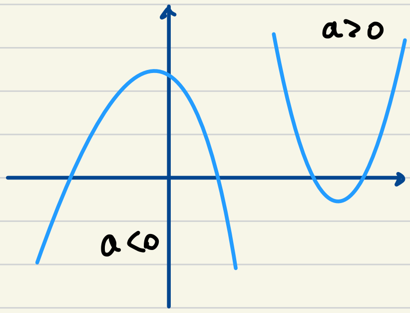
Second degré

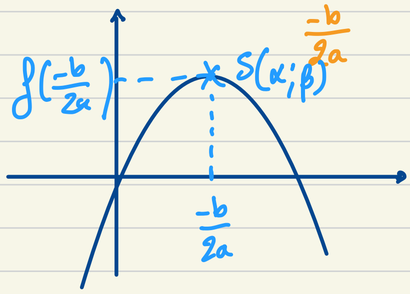
$f(x) = ax^2 + bx + c$ $\qquad a, b, c \in \mathbb{R}$, $a \neq 0$



. forme canonique :

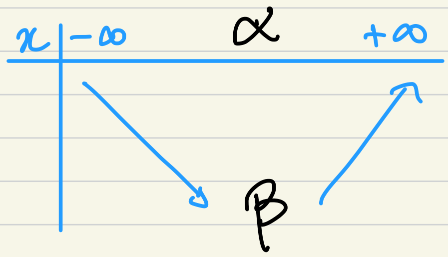
$f(x) = a(x - \alpha)^2 + \beta$

$\alpha = \frac{-b}{2a}$ $\qquad \beta = f\left(\frac{-b}{2a}\right)$

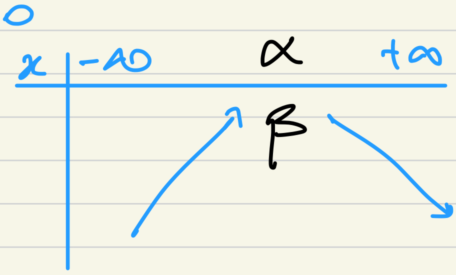


Variations

a>0



a<0



Application :

Donner le tableau de variation de $x^2 + 2x + 1$.

$= a(x+1)^2 + \beta$

$\alpha = -1 \qquad \beta = 0$

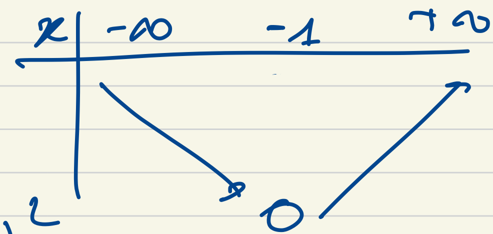
$a(x-\alpha)^2 + \beta$

$= 1\left(x - \left(\frac{-b}{2a}\right)\right)^2 + f\left(\frac{-b}{2a}\right)$

$= 1\left(x - \left(\frac{-2}{2\times 1}\right)\right)^2 + f\left(\frac{-2}{2\times 1}\right)$

$= 1(x+1)^2 + f(-1)$

$= (x+1)^2 + 0 \quad = (x+1)^2$



x^2+6x-8

x^2-5x+3

$$f(x) = a(x-\alpha)^2+\beta$$

$\alpha \to \frac{-b}{2a}$ $\quad \beta \to f\left(\frac{-b}{2a}\right)$

x^2+6x-8

$= 1(x-(-3))^2+(-17)$

$= (x+3)^2-17.$

$\alpha = \frac{-6}{2\times1} = -3$

$\beta = f(-3) = (-3)^2+6\times(-3)-8$

$= 9-18-8 = -17$

x^2-5x+3

$= 1(x-2{,}5)^2+\left(\frac{-13}{4}\right)$

$= (x-2{,}5)^2-\frac{13}{4}$

$\alpha = \frac{-(-5)}{2\times1} = \frac{5}{2} = 2{,}5.$

$\beta = \left(\frac{5}{2}\right)^2 - 5\times\frac{5}{2}+3 = \frac{25}{4}-\frac{50}{4}+\frac{12}{4}$

$= \frac{-13}{4} = -3{,}25.$

Resolution d'equation.

$2x^2+x = 0$

$x(2x+1) = 0$

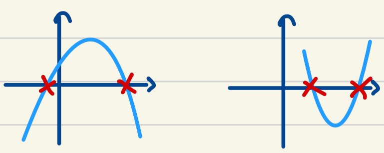
$x=0$ $\quad 2x+1=0$

$x = \frac{-1}{2}$

$ax^2+bx+c=0$

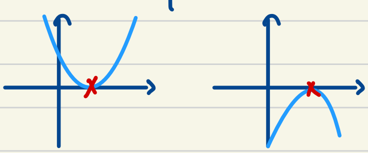
discriminant $(\Delta = b^2-4ac) \in \mathbb{R}$

$\Delta>0$ alors l'equation admets 2 solutions distinctes.



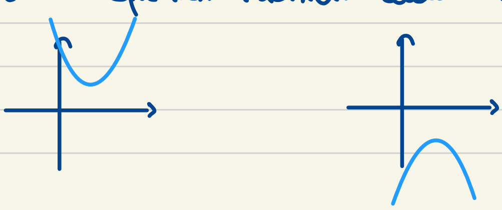
$x_1 = \frac{-b-\sqrt{\Delta}}{2a}$ $\quad x_2 = \frac{-b+\sqrt{\Delta}}{2a}$

$\Delta=0$ alors l'equation admets 1 unique solution.



$x_1 = \frac{-b}{2a}$

$\Delta<0$ alors l'equation n'admets aucune solution réelle.



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Application :

$x^2 - x - 6 = 0$

$\Delta = b^2 - 4ac$
$= (-1)^2 - 4 \times 1 \times (-6)$
$= 1 + 24$
$= 25$

$\Delta > 0$ 2 solutions

$x_1 = \frac{-b-\sqrt{\Delta}}{2a}$ $x_2 = \frac{-b+\sqrt{\Delta}}{2a}$

$x_1 = \frac{1-5}{2}$ $x_2 = \frac{1+5}{2}$

$= \frac{-4}{2}$ $= \frac{6}{2}$

$= \boxed{-2}$ $= \boxed{3}$

$x^2 + 2x - 3 = 0$

$\Delta = 2^2 - 4 \times 1 \times (-3)$
$\Delta = 4 + 12$
$\Delta = 16$

$x_1 = \frac{-2+\sqrt{16}}{2}$ $x_2 = \frac{-2-\sqrt{16}}{2}$

$x_1 = \frac{2}{2} = \boxed{1}$ $= \frac{-6}{2} = \boxed{-3}$

$x^2 - x + 2 = 0.$

$\Delta = (-1)^2 - 4 \times 1 \times 2$
$= 1 - 8 = -7 < 0$

pas de solutions.

$\frac{x^2 + 2x + 1}{x+1} = 2x - 1$

l'égalité n'est pas définie en (-1)

$x^2 + 2x + 1 = (2x-1) \times (x+1)$

$x^2 + 2x + 1 = 2x^2 + 2x - x - 1$

$x^2 + 2x + 1 = 2x^2 + x - 1$

$0 = 2x^2 - x^2 + x - 2x - 1 - 1$

$\boxed{0 = x^2 - x - 2}$

$\Delta = (-1)^2 - 4 \times 1 \times (-2)$
$= 1 + 8 = 9.$

$x_1 = \frac{1-\sqrt{9}}{2}$ $x_2 = \frac{1+\sqrt{9}}{2}$

$= \frac{-2}{2} = \cancel{\boxed{-1}}$ $= \frac{4}{2} = \boxed{2}$

on a un problème puisque le dénominateur s'annule en (-1) on retiendra seulement 2 comme solution.

$x^2 - 3x + 2 > 0$

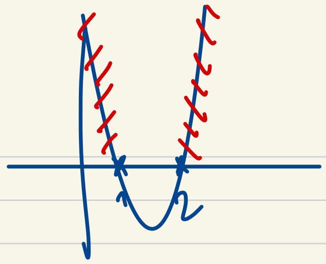
$\Delta = (-3)^2 - 4 \times 1 \times 2$
$= 9 - 8 = 1$

$x_1 = \frac{3+1}{2}$ $x_2 = \frac{3-1}{2}$

$= \boxed{2}$ $= \boxed{1}$

$x^2 + 4 \geq 0.$

$x \in \mathbb{R}.$



$x \in]-\infty; 1[\cup]2; +\infty[$

$x^2 - x + 1 < 0$.

$\Delta = (-1)^2 - 4 \times 1 \times 1$

$= 1 - 4 < 0$

$x \in \emptyset$ ensemble vide.