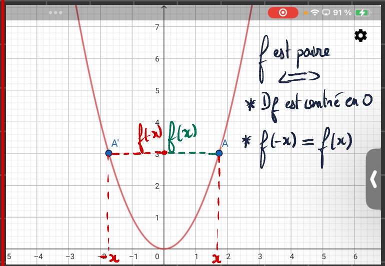
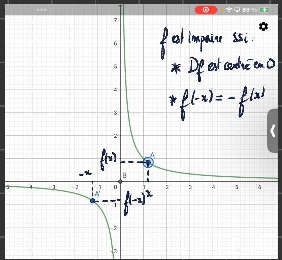
Mercredi 28 février 2024





$f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$

$f(x) = \frac{1}{x}$

$f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$

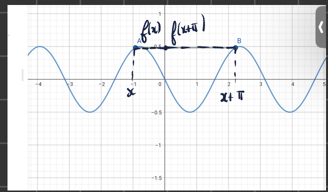
1) Soit $x \in \mathbb{R}$, centré en 0. $\quad 3 = -(-3)$

$$f(-x) = -\sin(-x)\cos(-x) = \sin(x)\cos(x) = -1\times(-\sin(x)\cos(x)) = -f(x)$$

f est impaire.

f est T-périodique ssi :

$$f(x+T) = f(x)$$



$$f(x+\pi) = -\sin(x+\pi)\cos(x+\pi) = \sin(x)\times(-\cos(x)) = -\sin(x)\times\cos(x) = f(x)$$

Donc f est π-périodique.

2. Soit $x \in \mathbb{R}$ $f(x) = -\sin(x)\cos(x)$.

Montrons que :

$$\boxed{f'(x) = 2\left(\sin(x) - \frac{1}{\sqrt{2}}\right)\left(\sin(x) + \frac{1}{\sqrt{2}}\right)}$$

ici $u(x) = -\sin(x)$ $\quad v(x) = \cos(x)$

$u'(x) = -\cos(x)$ $\quad v'(x) = -\sin(x)$

$$f'(x) = u'(x)\times v(x) + v'(x)\times u(x)$$

$$f'(x) = -\cos(x)\cos(x) + (-\sin(x))\times(-\sin(x))$$

$$f'(x) = -\cos^2(x) + \sin^2(x) = -(1-\sin^2(x)) + \sin^2(x) = \sin^2(x) - 1 + \sin^2(x) = 2\sin^2(x) - 1$$

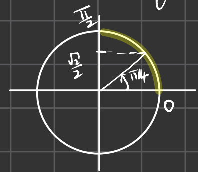
$$= \sin^2(x) - 1 + \sin^2(x) = 2\sin^2(x) - 1 = 2\left(\sin^2(x) - \frac{1}{2}\right)$$

$a^2 - b^2 = (a+b)(a-b)$

$$= 2\left(\sin^2(x) - \left(\frac{1}{\sqrt{2}}\right)^2\right)$$

$$= \boxed{2\underbrace{\left(\sin(x) + \frac{1}{\sqrt{2}}\right)}_{+}\left(\sin(x) - \frac{1}{\sqrt{2}}\right)}$$

3) Étudions le signe de $f'(x)$:

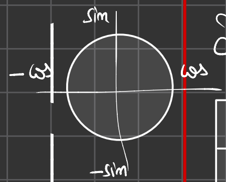


$\sin(x) - \frac{1}{\sqrt{2}} \geq 0$

$\sin(x) \geq \frac{1}{\sqrt{2}}$

$\sin(x) \geq \frac{\sqrt{2}}{2}$

$\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$



D'où :

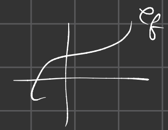
x	0		$\frac{\pi}{4}$		$\frac{\pi}{2}$
$2(\sin(x)+\frac{1}{\sqrt{2}})$		+		+	
$(\sin(x)-\frac{1}{\sqrt{2}})$		−	0	+	
$f'(x)$		−		+	
f	0	↘	$-\frac{1}{2}$	↗	0

$f(0) = -\sin(0)\cos(0)$ $\quad f(\frac{\pi}{2}) = -\sin(\frac{\pi}{2})\cos(\frac{\pi}{2})$

$f(0) = -0\times 1 = 0$ $\quad f(\frac{\pi}{2}) = -1\times 0 = 0$

$f(\frac{\pi}{4}) = -\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} = -\frac{2}{4} = -\frac{1}{2}$

4) Intersection I avec (Oy)



$f(0) = 0$.

$I(0;0)$.

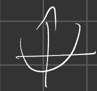
Intersect° J et K avec (Ox) :

$f(x) = 0$ $\quad A\times B = 0 \iff A = 0$ ou $B = 0$

$-\sin(x)\cos(x) = 0$.

$\sin(x) = 0$ $\quad \cos(x) = 0$

$x = 0$ $\quad x = \frac{\pi}{2}$



$I = K(0;0)$ et $J(\frac{\pi}{2};0)$

5) Éq° de la tangente à Cf en a.

T_a : $y = f'(a)(x-a) + f(a)$

T_0 : $y = f'(0)(x-0) + f(0)$.

$\boxed{T_0 : y = -x}$