plus de bonnes notes 10/03/24. Tle Spé maths.

$F(x) \longrightarrow f(x) \longrightarrow f'(x)$

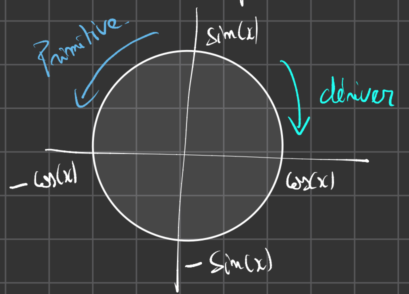
f° primitive — fonct° dérivée

$f(x) = x$

$f'(x) = 1 \qquad F(x) = \frac{1}{2}x^2 + C.$

$f(x) = \cos(x).$

$F(x) = \sin(x) \qquad f'(x) = -\sin(x).$



$f(x) = 5x^3.$

$F(x) = \frac{5}{4}x^4 \qquad f'(x) = 15x^2$

$y' = 2y$

$f(x) = Ce^{2x}.$

$y' : f'(x) = 2Ce^{2x}$

$2y : 2 \times f(x) = 2Ce^{2x}$

$y' = 2y + 4.$

$f(x) = Ce^{2x} - 2. \quad C \in \mathbb{R}.$

$f(0) = 3.$

$f(0) = 3.$

$Ce^{2\times 0} - 2 = 3.$

$C - 2 = 3.$

$\boxed{C = 5}$

$f(x) = 5e^{2x} - 2.$

Exercice 2 :

$f(x) = e^x - 2e^{-x}.$

$f(x) = e^x + 2 \times (-1) \times e^{-x}$

$f(x) = e^x + 2 \times u'(x)\, e^{u(x)}$ où $u(x) = -x.$

$F(x) = e^x + 2 \times e^{-x}.$

2) $f(x) = \dfrac{2x}{(x^2+3)^2} \qquad u(x) = x^2 + 3, \quad u'(x) = 2x$

$f(x) = \dfrac{u'(x)}{[u(x)]^2}$

$F(x) = -\dfrac{1}{x^2+3}$

3) $f(x) = \dfrac{x}{2\sqrt{x^2+1}} = \frac{1}{2} \times \frac{1}{2} \times \dfrac{2x}{\sqrt{x^2+1}}$

$f(x) = \frac{1}{4} \times \dfrac{2x}{\sqrt{x^2+1}} = \frac{1}{4} \times \dfrac{u'(x)}{\sqrt{u(x)}}$

$F(x) = \frac{1}{4} \times 2\sqrt{x^2+1} = \frac{1}{2}\sqrt{x^2+1}$

4) $f(x) = \dfrac{9x^2 - 3}{(x^3 - x)^2} = 3 \times \dfrac{3x^2 - 1}{(x^3 - x)^2}$

$u(x) = x^3 - x \qquad = 3\,\dfrac{u'(x)}{u^2(x)}$

$u'(x) = 3x^2 - 1$

$F(x) = 3 \times \dfrac{-1}{x^3 - x} = \dfrac{-3}{x^3 - x}.$

5) $f(x) = (4-x)(x^2 - 8x)^5 \qquad u' \times u^n$

$u(x) = x^2 - 8x \qquad f(x) = -\frac{1}{2}(2x-8)(x^2-8x)^5$

$u'(x) = 2x - 8$

$F(x) = -\frac{1}{2} \times \dfrac{(x^2-8x)^6}{6}$

$F(x) = -\frac{1}{12}(x^2 - 8x)^6$

n°6.

1) $y' + \sqrt{2}\,y = 0. \qquad F(\sqrt{2}) = 1.$

$y' = -\sqrt{2}\,y.$

$F(x) = Ce^{-\sqrt{2}x}$

$3 \times x = 7$

$x = \frac{7}{3}$

$Ce^{-\sqrt{2}\times\sqrt{2}} = 1$

$Ce^{-2} = 1.$

$C = \dfrac{1}{e^{-2}} = e^2.$

$F(x) = e^2 e^{-\sqrt{2}x}$

3) $\frac{1}{2}y' + y = \frac{1}{2}y - y' \qquad F(3) = \frac{1}{e}$

$\frac{1}{2}y' + y' = \frac{1}{2}y - y.$

$\frac{3}{2}y' = -\frac{1}{2}y.$

plus de bonnes notes

$f(x) = -2x-4 \quad f'(x) = 2$

$y' = -\frac{1}{3}y$

$F(x) = Ce^{-\frac{1}{3}x}$ où $C \in \mathbb{R}$

$F(3) = \frac{1}{e} \Leftrightarrow C \times e^{-\frac{1}{3}\times 3} = \frac{1}{e}$

$C \times e^{-1} = \frac{1}{e}$

$C \times \frac{1}{e} = \frac{1}{e}$

$\boxed{C = 1}$

$\boxed{F(x) = e^{-\frac{1}{3}x}}$

$2y' - y = 2x$

$f(x) = ax+b$ Soit sol° de E

$2a - (ax+b) = 2x$

Sp :

$2y' - y = 0$

$S_{G,H}$.

$S_{G,E} =$

$2y' - y = 2x$

Recherche de la sol° particulière affine :

$f(x) = ax+b$

$2a - (ax+b) = 2x$

$2a - ax - b = 2x$

$-ax + 2a - b = 2x + 0$

$\begin{cases} -a = 2 \\ 2a - b = 0 \end{cases} \Leftrightarrow \begin{cases} a = -2 \\ b = -4 \end{cases}$

$f(x) = -2x - 4$ solut° particulière de l'éq° générale

2°) Recherche de la sol° générale de l'éq° homogène.

$2y' - y = 0$

$2y' = y$

$y' = \frac{1}{2}y$

$g(x) = Ce^{\frac{1}{2}x}$

Sol° générale de l'éq° générale :

$h(x) = f(x) + g(x)$

$h(x) = -2x - 4 + Ce^{\frac{1}{2}x}$

$h(0) = -2$

$-2\times 0 - 4 + Ce^{\frac{1}{2}\times 0} = -2$

$-4 + C = -2$

$C = -2 + 4 = 2$

$\boxed{h(x) = -2x - 4 + 2e^{\frac{1}{2}x}}$

$a_0 x^0 + a_1 x^1 + \dots + a_m x^m$

$\| \quad \| = \quad \|$

$b_0 x^0 + b_1 x^1 + \dots + b_m x^m$