$$\int_1^e \frac{e}{x}\,dx = e\int_1^e \frac{1}{x}\,dx$$

$$= e\times[\ln(x)]_1^e$$

$$= e\times(\ln(e)-\ln(1))$$

$$= e\times 1 = e$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x+\sin^3(x)+\sin^5(x))\,dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3(x)\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5(x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 0 + 0$$

$$x_{n+1} = \int_0^1 \underbrace{t^{n+1}}_{u(t)}\,\underbrace{\cos(t)}_{v'(t)}\,dt$$

$u(t) = t^{n+1}$ $\quad$ $v(t) = \sin(t)$.

$u'(t) = (n+1)\,t^n$ $\quad$ $v'(t) = \cos(t)$.

$$x_{n+1} = \left[t^{n+1}\sin(t)\right]_0^1 - \int_0^1 (n+1)\times t^n \times \sin(t)\,dt$$

$$x_{n+1} = 1^{n+1}\times\sin(1) - (n+1)\times y_n$$

$$\boxed{x_{n+1} = \sin(1) - (n+1)\,y_n}$$

3) b) $x_{n+1} = -(n+1)\,y_n + \sin(1)$.

On suppose que $\lim\limits_{n\to+\infty} y_n = \pm\infty$

$\Rightarrow \lim\limits_{n\to+\infty} x_n = \pm\infty$

y_n ne diverge pas vers $\pm\infty$.

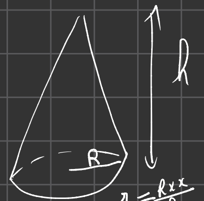
On suppose que (y_n) n'admet pas de limite.

$\Rightarrow (x_n)$ n'admet pas de limites

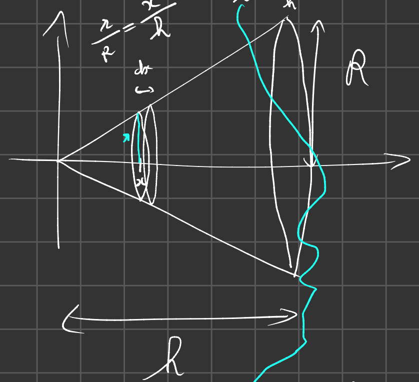
ce qui est faux.

D'où $\boxed{\lim\limits_{n\to+\infty} y_n = 0}$

4)



$V = \frac{1}{3}\times\pi\times R^2\times h$.



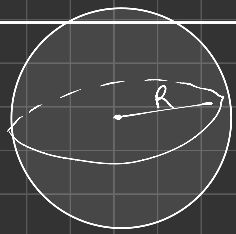
$$\int_0^h \pi\times r^2\times dx = \int_0^h \pi\times\left(\frac{R x}{h}\right)^2 dx$$

$$= \frac{\pi R^2}{h^2}\int_0^h x^2\,dx$$

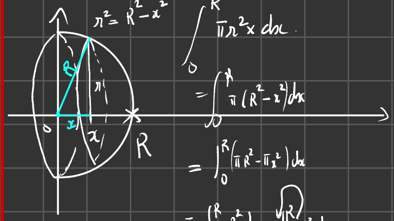
$$\frac{\pi R^2}{h^2}\left[\frac{x^3}{3}\right]_0^h$$

$$\frac{\pi R^2}{h^2\times 3}\times h^3$$

$$\boxed{\frac{1}{3}\times\pi R^2\times h}$$



$V = \frac{4}{3}\times\pi\times R^3$



$$\int_0^R \pi r^2\times dx$$

$$= \int_0^R \pi(R^2-x^2)\,dx$$

$$= \int_0^R (\pi R^2 - \pi x^2)\,dx$$

$$= \int_0^R \pi R^2\,dx - \int_0^R \pi x^2\,dx$$

$$= \left[\pi R^2 x\right]_0^R - \pi\left[\frac{x^3}{3}\right]_0^R$$

$$= \pi R^2\times R - \pi R^2\times 0 - \pi\frac{R^3}{3}$$

$$= \pi R^3 - \frac{1}{3}\pi R^3 = \boxed{\frac{2}{3}\pi R^3} = \frac{V}{2}$$

$$\boxed{V = \frac{4}{3}\pi R^3}$$