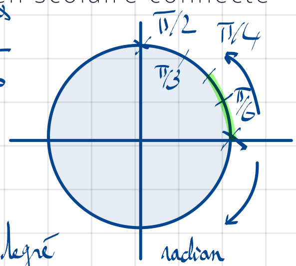
21/08/24 - Tale - Maths.

$\frac{4}{8} = \frac{8}{16} = \frac{1}{2}$




degré radian

$180° \longleftrightarrow \pi$ rad.

$90 \longleftrightarrow \frac{\pi}{2}$ rad.

$37° \longleftrightarrow \frac{37 \times \pi}{180}$

$\frac{\pi}{4} + \frac{2\pi \times 4}{1 \times 4} = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$

$\frac{9\pi}{4} + 2\pi$

La mesure principale d'un angle est la mesure de l'angle appartenant à l'intervalle $]-\pi ; \pi]$.

$\frac{182\pi}{3} = \frac{180\pi}{3} + \frac{2\pi}{3}$

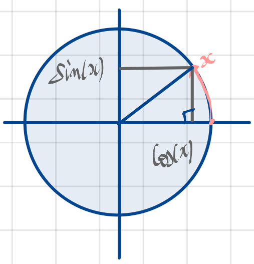
$= 60\pi + \frac{2\pi}{3}$

$= 30 \times 2\pi + \frac{2\pi}{3}$

$\frac{182}{8}$

Fonctions trigonométriques.

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$. $\cos(3)$




$\boxed{\tan(x) = \frac{\sin(x)}{\cos(x)}}$

$\cos(\pi)$.

Angles remarquables dans le cercle trigo.

| Angles | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|---|
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 | $-1$ |
| sin | 1 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 | 0 |

Propriétés : 1) $\forall x \in \mathbb{R}$, $-1 \leq \cos(x) \leq 1$

2) $\forall x \in \mathbb{R}$, $-1 \leq \sin(x) \leq 1$

3) $\forall x \in \mathbb{R}$ $\cos^2(x) + \sin^2(x) = 1$

4) $\cos(x+\pi) = -\cos(x)$

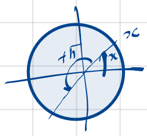
5) $\sin(x+\pi) = -\sin(x)$

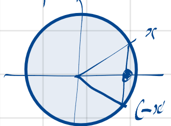
6) $\cos(-x) = \cos(x)$ paire $2\pi$-périodiques.

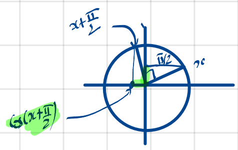
7) $\sin(-x) = -\sin(x)$ impaire.

8) $\cos\left(x+\frac{\pi}{2}\right) = -\sin(x)$.

9) $\sin\left(x+\frac{\pi}{2}\right) = \cos(x)$.



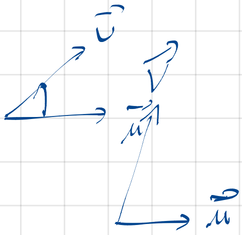



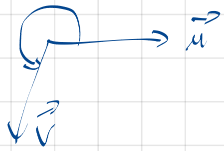




$\vec{u}\begin{pmatrix}1\\2\\3\end{pmatrix}$ $\vec{v}\begin{pmatrix}4\\5\\6\end{pmatrix}$.

$\vec{u}\cdot\vec{v} = 1\times4 + 2\times5 + 3\times6$.

$\vec{u}\cdot\vec{v} = \|\vec{u}\| \times \|\vec{v}\| \cos(\vec{u};\vec{v})$.



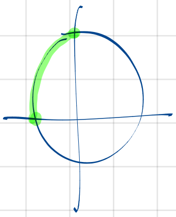

1) $x \in \left[\frac{\pi}{2} ; \pi\right]$, $\forall x \in \mathbb{R}$, $\cos^2(x) + \sin^2(x) = 1$.

$\cos^2(x) + \left(\frac{1}{4}\right)^2 = 1$.

$\cos^2(x) = 1 - \frac{1}{16}$.

$\cos^2(x) = \frac{15}{16}$.

$\cos^2(x) = \frac{15}{16}$.



$\cos(x) = \sqrt{\frac{15}{16}}$ ou $\cos(x) = -\sqrt{\frac{15}{16}}$

$\cos(x) = \frac{-\sqrt{15}}{4}$.

2) $x \in \left[-\frac{\pi}{3} ; \frac{\pi}{3}\right]$ et $\sin(x) = -0{,}6$

$\cos^2(x) + \sin^2(x) = 1$

$\cos^2(x) + (-0{,}6)^2 = 1$

$\cos^2(x) + 0{,}36 = 1$

$\cos^2(x) = 1 - 0{,}36$

$\cos^2(x) = 0{,}64$

$\cos(x) = \sqrt{0{,}64}$

$\boxed{\cos(x) = \frac{4}{5}}$ ou $\cos(x) = -\frac{4}{5}$

3) $x \in \left[-\frac{\pi}{2} ; 0\right]$ et $\sin(x) = -\frac{2}{3}$

$\cos^2(x) + \sin^2(x) = 1$

$\cos^2(x) + \left(-\frac{2}{3}\right)^2 = 1$

$\cos^2(x) = 1 - \frac{4}{9}$

$\cos^2(x) = \frac{5}{9}$

$\cos x = \sqrt{\frac{5}{9}}$ ou $\cos(x) = -\sqrt{\frac{5}{9}}$

$$\boxed{\cos x = \frac{\sqrt{5}}{3}}$$

Équation trigonométrique.

$x \in \mathbb{R}$.

$(E) : \cos(x) = a$. où $a \in \mathbb{R}$.

1) Si $a \notin ]-1;1[$, $(E)$ n'a pas de solutions réelles.

2) Sinon, i.e, $a \in [-1;1]$,

1°) trouver une valeur de $x$ telle que $\cos(x) = a$. On note cette valeur $\alpha$.
$(\cos(\alpha) = a$.

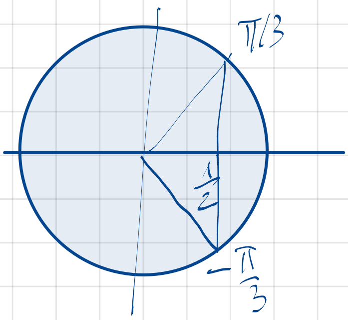
2°) $\begin{cases} x = \alpha + k \times 2\pi & k \in \mathbb{Z} = \{\dots -3; -2; -1; 0; 1; 2; 3 \dots\} \\ x = -\alpha + k' \times 2\pi. & k' \in \mathbb{Z}. \end{cases}$

Ex : $\cos(x) = \frac{1}{2}$.

$x + 5 = 8$.

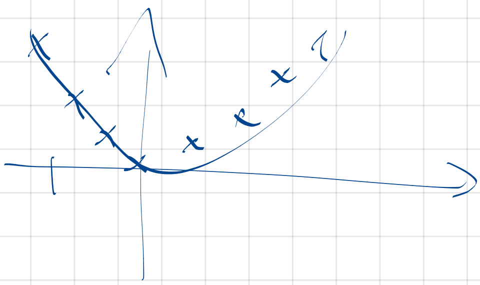
Or $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ d'où :

$\begin{cases} x = \frac{\pi}{3} + k \times 2\pi \text{ ou } k \in \mathbb{Z}. \\ x = -\frac{\pi}{3} + k' \times 2\pi \text{ ou } k' \in \mathbb{Z}. \end{cases}$

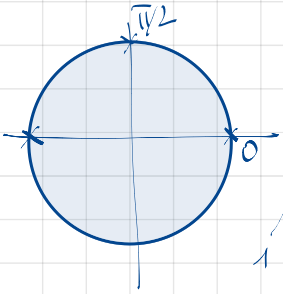



$f(x) = x^2$.

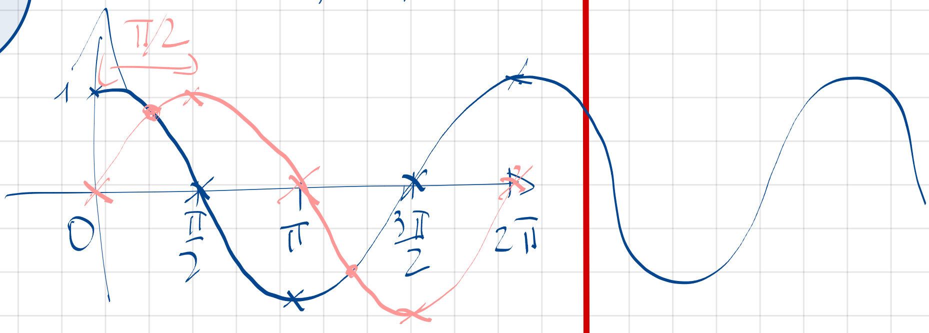
| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $9$ | $4$ | $1$ | $0$ | $1$ | $4$ | $9$ |



| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| --- | --- | --- | --- | --- | --- |
| $\sin(x)$ | $0$ | $1$ | $0$ | $-1$ | $0$ |




| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| --- | --- | --- | --- | --- | --- |
| $\cos(x)$ | $1$ | $0$ | $-1$ | $0$ | $1$ |




$\cos(x) = \sin(x)$