plus de bonnes notes
le soutien scolaire connecté

15/09/24.

4) Le nombre complexe conjugué.

Soit $z \in \mathbb{C}$ un nombre complexe tel que $z = a + ib$.
alors, le conjugué de z noté $\overline{z}$ est :

$$\overline{z} = a - ib.$$

Ex : $z = -3i + 2$
$\overline{z} = 3i + 2$.

Propriétés : 1) $\overline{\overline{z}} = z$ 2) $\overline{z+z'} = \overline{z} + \overline{z'}$

2) $\overline{z \times z'} = \overline{z} \times \overline{z'}$ 3) $\overline{\left(\frac{z}{z'}\right)} = \frac{\overline{z}}{\overline{z'}}$

3) $\overline{z^n} = \overline{z}^n \quad \forall n \in \mathbb{N}^*$.

Application : $z = \frac{1 \times (2-i)}{(2+i)\times(2-i)} = \frac{2-i}{4-i^2} = \frac{2-i}{5}$

$= \frac{2}{5} - \frac{1}{5}i = 0{,}4 - 0{,}2i$.

4) $z + \overline{z} = 2 \times Re(z)$. $\quad a+ib-(a-ib)$

5) $z - \overline{z} = 2i\, Im(z)$.

6) z est imaginaire pur $\Leftrightarrow z + \overline{z} = 0$

7) z est réel $\Leftrightarrow z = \overline{z}$.

Exo 4.

8) $z = \left(\frac{4-6i}{2-3i}\right)\left(\frac{1+3i}{3+2i}\right) = 2\left(\frac{1+3i}{3+2i}\right) = \frac{(2+6i)(3-2i)}{(3+2i)(3-2i)}$

$z = \frac{6-4i+18i+12}{3^2-(2i)^2} = \frac{6+14i+12}{9+4} = \frac{18+14i}{13} = \frac{18}{13} + \frac{14}{13}i$

$x \xmapsto{f} f(x)$

$n \xmapsto{(u_n)} u_n$

$\mathbb{C} \xrightarrow{f} \mathbb{C}$.
$z \longmapsto f(z) = \overline{z}$.

$z \longmapsto Z = z - 2\overline{z} + 2$.

1) $Z = x + iy - 2x(x-iy) + 2$.
$Z = x + iy - 2x + 2iy + 2$
$Z = -x + 2 + i \times 3y$.

2) $Z = 0$
$-x + 2 + i \times 3y = 0 + 0i$ $\quad z = x+iy$.

$\begin{cases} -x+2 = 0 \\ 3y = 0 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ y = 0 \end{cases}$

$\boxed{z = 2}$

II) 2) $z \longmapsto z' = \frac{5z-2}{z-1}$

$z' \in i\mathbb{R}$
↳ z' est imaginaire pur
↳ $Re(z') = 0 \Leftrightarrow$ M point d'un cercle privé d'un point

$z' = \frac{5(x+iy)-2}{x+iy-1} = \frac{(5x+5iy-2)(x-1-iy)}{(x-1+iy)\times(x-1-iy)}$

$z' = \frac{5x^2 - 5x - i5xy + i5xy - 5iy + 5y^2 - 2x + 2 + 2iy}{(x-1)^2 + y^2}$ $\quad x \neq 1$.

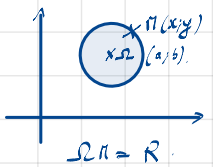
$z' = \frac{5x^2 - 7x + 5y^2 + 2}{(x-1)^2+y^2} + i\frac{-3y}{(x-1)^2+y^2}$.

$z' \in i\mathbb{R} \Leftrightarrow 5x^2 - 7x + 5y^2 + 2 = 0$.
$x^2 - \frac{7}{5}x + y^2 + \frac{2}{5} = 0$

$x^2 - 2 \times x \times \frac{7}{10} + \frac{49}{100} - \frac{49}{100} + y^2 + \frac{2}{5} = 0$.

$\left(x - \frac{7}{10}\right)^2 + (y-0)^2 = \frac{9}{100}$

Cercle centre $\Omega\left(0{,}7 ; 0\right)$ de rayon $0{,}3$.



$\Omega M = R$
$(x - x_\Omega)^2 + (y - y_\Omega)^2 = R^2$